Staple!

Ord: ___ ___

Abstract Algebra
MAS4301 3249

Class-X

Prof. JLF King
Wedn., 16Nov2022

Hello. Let F and R be the ***flip*** and ***rotation*** in the dihedral group $\mathbb{D}_N$, with $F^2=\mathbf{e}$, $R^N=\mathbf{e}$ and $RFRF=\mathbf{e}$. Use R^j and R^jF as the std. form of each element in $\mathbb{D}_N$. Symbol $\mathbb{Z}_N$ denotes the *cyclic gp* $(\mathbb{Z}_N, +, 0)$.

A perm of cyc-sig $\lceil 1^3, 5^2 \rfloor$ has three 1-cycles and two 5-cycles.

X1: Short answer. Show no work. Write LARGE.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\textbf{DNE} \neq \{\} \neq 0$.

___ ___ a) The IOP (Individual Optional Project), if you choose to do it, is due by 2PM on Friday, 09Dec2022, slid *completely* under my office door, Little Hall 402 (northeast corner of top floor) Circle: Yes Cool! Thanks

___ ___ b) From class, the group G of OP-isometries [Orientation-Preserving] of the cube is isomorphic to ....

Two colorings of the twelve *edges* of the cube using K colors, are ***equivalent*** IFF some OP-isometry carries one to the other. To compute $\mathcal{E}(K)$, the number of equiv-classes, fill in this table.

What isometry g?	$^{\#}$\{such g\}	$^{\#}$Fix(g) $= K^E$.	Cyc-sig, $^{\#}$[Edge-orbits under g].
Id	1	K^{12}	$\lceil 1^{12} \rfloor$, 12
FaceRot 90°			
FaceRot 180°			
VertexRot 120°			
EdgeRot 180°			

And $\mathcal{E}(K) = \ldots \cdot \left[K^{12} + \ldots\ldots\right]$.

___ ___ c) A finite group Γ acts on a finite set Ω. *Then...*

The number of Γ-orbits divides $|\Gamma|$: *T* *F*

Cardinality of *each* Γ-orbit divides $|\Gamma|$: *T* *F*

___ ___ d) The dihedral group $G := \mathbb{D}_{17}$ acts on the edges of a 17-gon. Let C be the number of "*really different*" red/blue Colorings of the edges. Let D be the # of "*really different*" ways of assigning a Direction to each edge. Then (circle one relation) $C > D$ $C = D$ $C < D$.

OYOP: *In grammatical English* ***sentences****, write your essays on every 2nd line* (usually)*, so I can easily write between the lines.*

X2: Define what it means for a map $\alpha: G \to G$ to be an ***automorphism*** of G.

Define groups $\text{Aut}(G)$ and $\text{Inn}(G)$, specifying the gp-operation. Prove that $\text{Inn}(G) \lhd \text{Aut}(G)$.

X3: For subgps $H, K \subset G$, suppose that $K \lhd G$. Prove that the set-product HK is a sub*group* of G. [*Hint:* First show that $KH = HK$.]

End of Class-X

X1: ___ ___ ___ 105pts

X2: ___ ___ 65pts

X3: ___ ___ 35pts

Total: ___ ___ ___ 205pts

NAME: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor or TA."*

Signature: ....