Staple!

Ord: ___ ___

Abstract Algebra
MAS4301 3249

Class-W

Prof. JLF King
Wednesday, 26Oct2022

Hello. Let F and R be the ***flip*** and ***rotation*** in the dihedral group $\mathbb{D}_N$, with $F^2=e$, $R^N=e$ and $RFRF=e$. Use R^j and R^jF as the std. form of each element in $\mathbb{D}_N$. Symbol $\mathbb{Z}_N$ denotes the *cyclic gp* $(\mathbb{Z}_N, +, 0)$.

A perm of cyc-sig $\lceil 1^3, 5^2 \rfloor$ has three 1-cycles and two 5-cycles.

W1: Short answer. Show no work. Write LARGE.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ ___ **a** For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course (Circle):

Yes **True** **Darn tootin'!**

___ ___ **b** In dihedral group $G := \mathbb{D}_{24}$ generated by flip F and rotation R, consider the subgroup $H := \langle F, R^4 \rangle_G$. The index of H in G is $|G : H| =$ ________.

___ ___ **c** Binomial $\binom{6}{2} =$ ________. So a $\lceil 3^6 \rfloor$-perm $\beta \in \mathbb{S}_{18}$ has ________ many square-roots of sig$=\lceil 6^1, 3^4 \rfloor$. A *cube*-root of β has signature $= \lceil$ ________ $\rfloor$.

___ ___ **d** In $\mathbb{S}_{13}$, the maximum possible order of an element is $\text{MaxOrd}(\mathbb{S}_{13}) = \text{LCM}($ ________ $) =$ ________.

___ ___ **e** Sequence $[1,2,3,4,5,6,7]$ is fed into a shuffling machine, then again, then again. The resulting perm, $\gamma := \alpha^3 \in \mathbb{S}_7$, is sequence $[2,4,1,6,3,7,5]$. So just *one* shuffle put $[1,2,3,4,5,6,7]$ in order

____ ____ ____ ____ ____ ____ ____

OYOP: *In grammatical English* ***sentences****, write your essays on every 2nd line* (usually)*, so I can easily write between the lines.*

W2: For $N = 3,4,5,\ldots$, prove each perm β in $\mathbb{A}_N$ [the N^{th} alternating gp] can be written as a composition [L-to-R] of 3-cycles. [The cycles need not be disjoint.]

Define $\alpha := (0,1,2,3,4,5)(6,7)$ in $\mathbb{A}_8$. As a product of 2-cycles, $\alpha =$ ________.

[The cycles need *not* be disjoint.] As a product of 3-cycles, $\alpha =$ ________.

W3: For groups $G \supset H$, formally state Lagrange's theorem. *Carefully* prove Lagrange's thm, starting with "*Proof:*".

End of Class-W

W1:	___ ___ ___	100pts
W2:	___ ___	55pts
W3:	___ ___	40pts
Total:	___ ___ ___	195pts

NAME: ________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor or TA."*

Signature: ________