Ord: ___ ___

Plex
MAA4402 3509

Class-U

Prof. JLF King
Wedn, 10Apr2024

NB. For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

Let **holom** abbreviate "holomorphic", and **harm.fnc** abbreviate "harmonic function".

U1: Short answer. Show no work.

___ ___ **a** EoS 2024 *Games Party*, from 12:50 pm – 4:30 pm, will take place at *Pascal's Cafe* on Wedn, 24Apr.

Circle   **Yes!**   **True!**   **I'll-bring-a-game!**

___ ___ **b** The visual representation of $\mathbb{C}$ is sometimes called "the ? plane", where ? is Circle: **Unreal** **Higher** **Snakes-on-a** **Argand** **Krypton** **Rayon** **Xenon** **Euler** **Goursat** **Liouville** **No-need-to-x** **y-com** **Air** **Sea** **De** **Rain-in-Spain-stays-mainly-on-the** .

___ ___ **c** Let

$$f(z) := z^4 \exp(2/z).$$

Then residue $\operatorname{Res}(f, 0) =$ ____________ .

[*Hint: Write the PS for* e^w, *then plug in* $2/z$ *for* w. *Multiply the resulting Laurent Series by* z^4. *You may use the factorial symbol in expressing your answer. Then simplify your answer.*]

___ ___ **d** Let C be SCC $\operatorname{Sph}_7(0)$, a circle of radius 7. Then

$$\oint_C \frac{\cos(2z)}{[z-5]^4}\, dz = \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

[Answer may be written as a product, using powers and factorials and $\sin()$ and $\cos()$.]

___ ___ **e** Define $f(x+\mathbf{i}y) := xy + \mathbf{i}x$. Let L be the line-segment from the origin to $2+\mathbf{i}$. Then $\int_{\mathsf{L}} f(z)\, dz =$ ____________ .

___ ___ **f** $\displaystyle\sum_{n=3}^{\infty} \left[\frac{1+2\mathbf{i}}{3}\right]^n =$ ________ $+ \left[\mathbf{i} \cdot \text{\_\_\_\_\_\_\_\_}\right]$.

U2: Short answer. Show no work.

___ **g** For a SCC C, suppose fncs f, g, analytic on $\widehat{\mathsf{C}}$, satisfy that $|f(z)| \geq |g(z)|$ for every $z \in \mathsf{C}$. If $f+g$ has fewer zeros in $\mathring{\mathsf{C}}$ than f does, then there must exist a point $w \in \mathsf{C}$ such that ____________ .

___ **h** Let $f(z) := z^5 + 3z^4 + 6z$, and $\mathsf{C}_r := \operatorname{Sph}_r(0)$. Our f has ________ zeros enclosed by C_1, and ________ zeros in annulus $\mathbf{A} := \operatorname{Ann}_2^1(0)$.

___ **i** In ball $\operatorname{Bal}_1(0)$, there are ________ solutions to

$$2z^9 - z^6 - 7z^3 + z = 2. \quad [\textit{Hint:} \text{ Rouché's thm.}]$$

___ **j** Gamma fnc: $\mathbf{\Gamma}(7) =$ ________ and $\mathbf{\Gamma}(\frac{7}{2}) =$ ________

For all real $x > 1$, our $\mathbf{\Gamma}()$ function satisfies recurrence relation $\mathbf{\Gamma}(x) =$ ____________ .

U1:	___ ___ ___	120pts
U2:	___ ___	95pts
Total:	___ ___ ___	215pts

NAME: ____________________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____________________