Staple!

Ord: ___ ___

Plex
MAA4402 3509

Class-T

Prof. JLF King
Wednesday, 06Mar2024

NB. For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

Let **holom** abbreviate "holomorphic", and **harm.fnc** abbreviate "harmonic function".

T1: Short answer. Show no work.

___ ___ a) For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics [e.g, my NUMBER THEORY AND CRYPTOGRAPHY], or graduate course Circle:

Yes **True** **Darn tootin'!**

___ ___ b) Prof. King has senior-citizen eyes, and *cannot read small handwriting.* Circle **True!** **Yes!** ***Who??***

___ ___ c) $\text{Res}\left(\frac{e^{2z}}{[z-5]^4}, z=5\right) =$ ______________.

___ ___ d) Write holomorphic $h(x+\mathbf{i}y)$ as $\mathsf{u}(x,y)+\mathbf{i}\mathsf{v}(x,y)$.

Then: Sum $6\mathsf{u} + 3\mathsf{v}$ is harmonic: *AT* *AF* *Nei*

A prod. of two harm.fncs is harmonic. *AT* *AF* *Nei*

If functions $f, g: \mathbb{R}\times\mathbb{R}\to\mathbb{R}$ are each harmonic, then sum $H(x+\mathbf{i}y) := f(x,y) + \mathbf{i}g(x,y)$ is holom. *AT* *AF* *Nei*

Fnc $\alpha(x+\mathbf{i}y) := x^2 + [2xy\mathbf{i}] - y^2$ is holomorphic. *T* *F*

___ ___ e) Power series $f(z) = \sum_{n=3}^{\infty} \frac{[3z]^n}{n+7}$ has

RoC= ________. And BoC= ______________.

OYOP: *In grammatical English sentences, write each essay on every 2nd line* (usually)*, so that I can easily write between the lines.*

T2: Let $f(z) := z^4 + 5z^2 + 4$, mapping $\mathbb{C}\to\mathbb{C}$. Reciprocal $H(z) := 1/f(z)$ has, in the upper half-plane, two poles $\mathbf{p}$ and $\mathbf{q}$, where $\mathbf{p}$ lies closer to the origin than $\mathbf{q}$.

So $\text{Res}(H, \mathbf{p})=$ ________ *and* $\text{Res}(H, \mathbf{q})=$ ________.

Our D-contour technique applies to H.

Thus $J := \int_{-\infty}^{+\infty} \frac{1}{x^4 + 5x^2 + 4}\,dx =$ ______________.

Give a full proof, illustrated with large, labeled diagrams.

T3: Below, $h: \mathbb{C}\to\mathbb{C}$, and $\mathsf{S} \subset \mathbb{C}$ is a SCC, and $\mathbf{w} \in \mathbb{C}$ is an *appropriate* point.

α) Detailing the conditions needed on h, S and $\mathbf{w}$, *carefully* state the Cauchy Integral Formula Theorem.

β) Recall the Cauchy Homotopy Thm: *Suppose closed-curves* S *and* R *are homotopic in an open set on which a fnc* f *is holomorphic. Then* $\oint_{\mathsf{S}} f = \oint_{\mathsf{R}} f$.

Use the above CHT to give a formal proof of the Cauchy Integral Formula Theorem. Also draw LARGE pictures showing the ideas in the proof.

T1:	___ ___ ___	105pts
T2:	___ ___	55pts
T3:	___ ___	55pts
Total:	___ ___ ___	215pts

NAME: ______________________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______________________