

Plex
MAA4402 3509

Class-S

Prof. JLF King
Wednesday, 07Feb2024**NB.** For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{ \} \neq 0$.Let **holom** abbreviate “holomorphic”, and **harm.fnc** abbreviate “harmonic function”.**S1:** Short answer. Show no work.

a

Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is **March 30, 2023**.

Circle: Yes True Résumé material!

b

Write $\cos(-2i)$, which is real,
ITOf $\exp()$ and *finite*
add/sub/mul/div: $\cos(-2i) =$ _____
And $\cos(-2i)$ lies in circle the correct interval
 $(-\infty, -\frac{1}{5}]$ $(-\frac{1}{5}, \frac{1}{5}]$ $(\frac{1}{5}, 2]$ $(2, 5]$ $(5, 15]$ $(15, 45]$ $(45, \infty)$

c

Distance $|e^{i[\pi/4]} - 2i| =$ _____.

d

The point $\omega := -3 + 2i$ corresponds, under stereographic projection, to (x, y, z) on the RS, where $x =$ _____, $y =$ _____, $z =$ _____.

e

Cross-ratio $[z, 2-i, \infty, 3i] = \frac{az+b}{cz+d}$ where
 $a =$ _____, $b =$ _____, $c =$ _____, $d =$ _____.

f

Fnc $u(x, y) := 2xy + x$ has
harmonic conjugate $v(x, y) =$ _____.In terms of $z = x + iy$, function $f := u + iv$ is $f(z) =$ _____. [ITOf z ; no x nor y .]OYOP: In grammatical English *sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.For a fnc h of form $h(z) = \frac{az+b}{cz+d}$, let $\text{Det}(h)$ mean $\text{Det} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. E.g,

$$g(z) = \frac{[3-i] - 2z}{[4+zi] \cdot 2}$$

has $\text{Det}(g)$ meaning the determinant of $\begin{bmatrix} -2 & 3+i \\ 2i & 8 \end{bmatrix}$.**S2:** Define LFT $V_P(z) := [z - P]/[1 - \bar{P}z]$, where complex number P has $|P| \neq 1$.Thus $\text{Det}(V_P) =$ _____.**Essay:** The unit-circle is $S := \{u \in \mathbb{C} \mid |u| = 1\}$. Prove that V_P maps S into S . [Hint: Complex number ω is in S IFF product $\omega\bar{\omega}$ equals *What?*]Map V_P sends the center of S to itself: T F**S1:** _____ 135pts**S2:** _____ 60pts**Total:** _____ 195pts

NAME: _____

HONOR CODE: *“I have neither requested nor received help on this exam other than from my professor.”*

Signature: _____