

Plex MAA4402 3509 Class-S Prof. JLF King Wednesday, 07Feb2024

NB. For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.
Let **holom** abbreviate “holomorphic”, and **harm.fnc** abbreviate “harmonic function”.

S1: Short answer. Show no work.

a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is **March 30, 2023**.

Circle: Yes True **Résumé material!**

b Write $\cos(-2i)$, which is real,
ITOf $\exp()$ and *finite*
add/sub/mul/div: $\cos(-2i) =$
And $\cos(-2i)$ lies in [circle the correct interval]
 $(-\infty, -\frac{1}{5})$ $(-\frac{1}{5}, \frac{1}{5})$ $(\frac{1}{5}, 2]$ $(2, 5]$ $(5, 15]$ $(15, 45]$ $(45, \infty)$

c Distance $|e^{i[\pi/4]} - 2i| =$

d The point $\omega := -3 + 2i$ corresponds, under stereographic projection, to (x, y, z) on the RS, where
 $x =$, $y =$, $z =$

e Cross-ratio $[z, 2-i, \infty, 3i] = \frac{az+b}{cz+d}$ where
 $a =$, $b =$, $c =$, $d =$

f Fnc $u(x, y) := 2xy + x$ has
harmonic conjugate $v(x, y) =$
In terms of $z = x + iy$, function
 $f := u + iv$ is $f(z) =$ [ITOf z ; no x nor y .]

OYOP: In *grammatical English sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.

For a fnc h of form $h(z) = \frac{az+b}{cz+d}$, let $\text{Det}(h)$ mean $\text{Det}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$. E.g.,
$$g(z) = \frac{\overline{[3-i]} - 2z}{[4+zi] \cdot 2}$$

has $\text{Det}(g)$ meaning the determinant of $\begin{bmatrix} -2 & 3+i \\ 2i & 8 \end{bmatrix}$.

S2: Define LFT $V_P(z) := [z - P]/[1 - \overline{P}z]$, where complex number P has $|P| \neq 1$.

Thus $\text{Det}(V_P) =$

Essay: The unit-circle is $S := \{u \in \mathbb{C} \mid |u| = 1\}$. Prove that V_P maps S into S . [Hint: Complex number ω is in S IFF product $\omega\overline{\omega}$ equals *What?*]

Map V_P sends the center of S to itself: T F

S1: ____ 135pts

S2: ____ 60pts

Total: ____ 195pts

NAME:

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: