Staple!

Ord: ___ ___

Linear Algebra MAS4105 *3247* — **Class-R** — Prof. JLF King, Wednesday, 27Sep2023

Hello. Use L_M for the lefthand-action of matrix M. Use B^t for the transpose of B. When working over $\mathbb{Z}_p$, state answers using *symmetric residues*, e.g, in $\mathbb{Z}_{13}$, answers should lie in [-6 .. 6].

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

R1: Show no work.

___ ___ z) Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea.* A ten-page essay is fine. Date for the emailed-PDF is Monday, 25Mar.2024.

Circle: **Yes** **True** ***Résumé material!***

___ ___ a) VS $\mathbf{V} := \mathrm{MAT}_{4\times4}(\mathbb{R})$ is a 16-dim'al $\mathbb{R}$-VS. Define linear-trn $D\colon \mathbf{V}\to\mathbf{V}$ by $D(\mathsf{M}) := \mathsf{M} + \mathsf{M}^\mathsf{t}$. Then Nullity$(D) =$ ________. And Rank$(D) =$ ________.

The trn $U\colon \mathbf{V}\to\mathbf{V}$ by $U(\mathsf{M}) := \mathsf{M}^2$ is: Circle best

Linear Affine (but not linear) Not-affine

___ ___ b) Shear the plane *vertically*, sending $\mathbf{e}_1$ to $\mathbf{e}_1 + 3\mathbf{e}_2$, followed by the *horizontal* shear which sends $\mathbf{e}_2$ to $-2\mathbf{e}_1 + \mathbf{e}_2$. Let S be the 2×2 matrix whose lefthand action is the preceding composition of shears.

Then $\mathsf{S} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

___ ___ c) Let $\mathsf{T}\colon\mathbb{R}^2\to\mathbb{R}^2$ by $\mathsf{T}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) := \begin{bmatrix} 3x-y \\ 2x+6y \end{bmatrix}$. W.r.t ordered-basis $\mathcal{B} := \left(\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \end{bmatrix}\right)$, let $\mathsf{M} := [\![\mathsf{T}]\!]_\mathcal{B}^\mathcal{B}$. Then $\mathsf{M} = \mathsf{R}\mathsf{T}\mathsf{R}^{-1}$,

where $\mathsf{R} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$, $\mathsf{M} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$.

___ ___ d) In each blank below, write either "there exist" or "for all", Circle one of the underlined scalar-pairs, and Circle a phrase.

Assertion $\mathrm{Spn}(\mathbf{v},\mathbf{w}) \supset \mathrm{Spn}(\mathbf{x},\mathbf{y})$ *means:*

"________ scalars $\underline{a,b}$ | $\underline{c,d}$ (*st.* | *we have that* | *and*)

________ scalars $\underline{a,b}$ | $\underline{c,d}$ (*st.* | *we have that*)

$a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}$."

R2: OYOP: Essay: *Write on every* ***second*** *line, so that I can easily write between the lines.*

i) Consider a lin-trn $\mathsf{T}\colon\mathbf{X}\to\mathbf{U}$ between finite-dimensional VSes. Distinguishing between zero-vectors $\vec{\mathbf{0}}_\mathbf{X}$ and $\vec{\mathbf{0}}_\mathbf{U}$, give, using *set-builder notation*: *A* ***formal definition*** *of* Range(T). And: *A* ***formal defn*** *of* Nul(T).

ii) State the Rank+Nullity theorem for $\mathsf{T}\colon\mathbf{X}\to\mathbf{U}$, first *defining* terms Rank(T) and Nullity(T).

iii) Give a careful proof of the Rank+Nullity thm for $\mathsf{T}\colon\mathbf{X}\to\mathbf{U}$. Also: Use good, large *pictures* to illustrate the ideas in the proof.

End of Class-R

R1:	___ ___ ___	125pts
R2:	___ ___	95pts
Total:	___ ___ ___	220pts

NAME: ________________________________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________