Plex
MAA4402

Quizzes P

Friday 20Jan2023

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** *and* $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b\,..\,c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e}\,..\,2\pi] = \{3, 4, 5, 6\} = [3\,..\,6] = (2\,..\,6]$. We allow b and c to be $\pm\infty$; so $(-\infty\,..\,-1]$ is $\mathbb{Z}_-$. And $[-\infty\,..\,-1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathbf{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2] \cdot \mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2] \cdot \mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※ ="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation. A *sequence* $\vec{\mathbf{x}}$ abbreviates $(x_0, x_1, x_2, x_3, \dots)$. For a set Ω, expression "$\vec{\mathbf{x}} \subset \Omega$" means $[\forall n\colon x_n \in \Omega]$. Use $\mathrm{Tail}_N(\vec{\mathbf{x}})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \dots)$$

of $\vec{\mathbf{x}}$. Given a fnc $f\colon \Omega \to \Lambda$ and an Ω-sequence $\vec{\mathbf{x}}$, let $f(\vec{\mathbf{x}})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \dots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, then, let $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} = [n \mapsto [x_n + y_n]] .$$

Similarly, $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

Plex notation. Let ***SCC*** mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{\mathsf{C}}$ be the (open) region C encloses, and let $\widehat{\mathsf{C}}$ mean C together with $\mathring{\mathsf{C}}$. So $\widehat{\mathsf{C}}$ is $\mathsf{C} \sqcup \mathring{\mathsf{C}}$; it is automatically simply-connected and is a closed bounded set.

Plex [2023g] quizzes so far...

P1: Fri. 20 Jan Blanks∈ℝ. So $\frac{1}{2+3\mathbf{i}} = \underline{\hspace{3cm}} + \mathbf{i}\cdot\left[\underline{\hspace{3cm}}\right]$.

Thus $\operatorname{Im}\left(\frac{5-\mathbf{i}}{2+3\mathbf{i}}\right) = \underline{\hspace{4cm}}$.

By the way, $|5-3\mathbf{i}| = \underline{\hspace{4cm}}$.

P2: Mon. 23 Jan Set $w := \left[2+\mathbf{i}\right]^2$. So $|w| = \underline{\hspace{5cm}}$.

Also, $\operatorname{Im}(\mathsf{e}^w) = \sin(A)\cdot \mathsf{e}^B$, for real numbers $A = \underline{\hspace{4cm}}$ and $B = \underline{\hspace{4cm}}$.

P3: Wed. 25 Jan Number $6 \cdot \exp\left(\mathbf{i} \cdot \frac{5\boldsymbol{\pi}}{3}\right)$ equals $x + y\mathbf{i}$ for reals

$x=$ .... and $y=$ ....

P4: Mon. 30 Jan Number $\left[\mathbf{i} + \sqrt{3}\right]^{70} = x + \mathbf{i}y$, for real numbers

$x=$ .... and $y=$ ....

[*Multiplying complexes multiplies their moduli (absolute-values), and adds their angles.*]

P5: Fri. 03 Feb On a set Ω, a ***metric*** is a map $\mathsf{d}\colon\Omega\to[0,\infty)$ such that $\forall\, w,x,y,z\in\Omega$:

MS1: ________________________________.

MS2: ________________________________.

MS3: ________________________________.

Class-A. Wed. 15 Feb

In-class closed-book Open-Brain exam.

Please bring lined-paper for computation. You may also want to bring colored pens/pencils for diagrams.

P6: Mon. 20 Feb Multinomial coefficient $\binom{9}{4,2,3}$ = ____________ = ____________.

[Write your answer as a product of binomial coeffs, then compute the product as a single integer,]

P7: Wed. 22 Feb *Am I in class today?*

circle one *"Yes!"* *"Of course!"*

"I wouldn't miss it for the world!"

P8: Fri. 24 Feb The coeff of x^7y^{12} in $[5x+y^3+1]^{30}$ is ________________________________.

[Write your answer as a product of powers and a multinomial. Optionally, you can expand the multinomial as a product of binomials.]

Bonus-A: Mon. 27 Feb In $\left[5x^2 + 4y + z^3 + 7\right]^{20}$, compute these coeffs:

$\mathrm{Coeff}(x^6 z^8) =$ ______________________.

$\mathrm{Coeff}(y^5 z^6) =$ ______________________

[An integer, or a product of powers and multinomial-coeffs.]

Map $\gamma\colon[0,1]\to\mathbb{C}$ is the constant speed parametrization of the line-segment from 3 to $\mathbf{i}$. So $\gamma(t)=$ ______________________.

Thus $J := \int_\gamma z\,\mathrm{d}z$ equals ______________________.

Class-B. Wed. 08Mar2023

In-class closed-book Open-Brain exam.
Please bring lined-paper for computation. You may also want to bring colored pens/pencils for diagrams.

Bonus-B: Fri. 10 Mar Coeff of $x^5 y^{18}$ in $\left[x + 1 + 3y\right]^{30}$ is ______.

[You may leave your answer as a product of *posints*, or you may multiply-out.]

Define $f(x + \mathbf{i}y) := xy + \mathbf{i}y$. Let L be the line-segment from $\mathsf{P} := 2\mathbf{i}$ to
$\mathsf{Q} := 1$. Then $\int_{\mathsf{L}} f(z)\,\mathrm{d}z =$ __________________________.

P9: Mon. 20 Mar. Recall: "The arclength-average on a circle, of a holomorphic function, is its value at the center."

With $\mathsf{C} := \mathrm{Sph}_2(3\mathbf{i})$ and $f(z) := z^2$,

integral $\oint_{\mathsf{C}} f(z)\,|\mathrm{d}z| =$ ________. The due-date for this

year's *Robert Long Essay Competition (RLEC)* is Thurs., 30 Mar., with a PDF emailed to Prof. K. **True!** **Yes!**

PA: Wed. 22 Mar Numeric sequence $\vec{\mathbf{a}} = (a_0, a_1, a_2, \ldots)$ is a ***Cauchy sequence*** if:

PB: Fri. 07 Apr Carefully state Rouche's theorem.

Bonus-C: Fri. 14 Apr Let $f(z) := z^5 + 3z^4 + 6z$, and $\mathsf{C}_r := \mathrm{Sph}_r(0)$. Our f

has ________ zeros enclosed by C_1, and ________ zeros

in annulus $\mathbf{A} := \mathrm{Ann}_2^1(0)$.

End of semester; looking forward to our Games Party!