Generalized parentheses. **Common:**
( ) *Parentheses.* [ ] *Brackets.* ⟨ ⟩ *Angle-brackets.*
{ } *Braces.* | | *Vertical bars* (*abs.value, cardinality*).

Less common: ⌊ ⌋ *Floor.* ⌈ ⌉ *Ceiling.* ‖ ‖ *Norm-of.*
⟦ ⟧ *Double-bracket.* ⟪ ⟫ *Double-angle-bracket.*

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** *and* $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[\mathrm{e} \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$. And $[-\infty \,..\, -1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2\mathrm{i}| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', *but* pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{\mathrm{d}v}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = \mathrm{e}^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(\mathrm{e}^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2]\cdot\mathrm{e}^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3t\mathrm{e}^{2t} + [t^2]\cdot\mathrm{e}^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※ ="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation. A *sequence* $\vec{\mathbf{x}}$ abbreviates $(x_0, x_1, x_2, x_3, \dots)$. For a set Ω, expression "$\vec{\mathbf{x}} \subset \Omega$" means $[\forall n\colon x_n \in \Omega]$. Use $\mathrm{Tail}_N(\vec{\mathbf{x}})$ for the subsequence

$$(x_N,\ x_{N+1},\ x_{N+2}, \dots)$$

of $\vec{\mathbf{x}}$. Given a fnc $f\colon\Omega\to\Lambda$ and an Ω-sequence $\vec{\mathbf{x}}$, let $f(\vec{\mathbf{x}})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \dots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$, then, let $\vec{\mathbf{x}} + \vec{\mathbf{y}}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{\mathbf{x}} + \vec{\mathbf{y}} \quad = \quad [n \mapsto [x_n + y_n]] \,.$$

Similarly, $\vec{\mathbf{x}} \cdot \vec{\mathbf{y}}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

SeLo [2023t] quizzes

(The lowest MQ score is dropped. In consequence, there is *no make-up* for the first missed MQ.)

[Monday, 04Sep2023: Labor Day, *No class*]

P1: Wed. 06 Sep Write $N := 36300$ as a PoPP [Product of Prime Powers],

$N \overset{\text{PoPP}}{=\!=}$ ________ . Hence the number of (positive) divisors of N is $\boldsymbol{\tau}(N) =$ ________ .

A function $f{:}\mathbb{R}\to\mathbb{R}$ is an ***involution*** IFF

________ .

[Don't forget the appropriate $\forall$ or $\exists$ quantification.]

P2: Fri. 08 Sep [With $\mathcal{P}()$ the *powerset* operator, let $S := $ 3-stooges.] Then $\left|\mathcal{P}(S)\right| =$ ____ and $\left|\mathcal{P}\big(\mathcal{P}(S)\big)\right| =$ ________ .

Math-Greek alphabet: Please write the **two** missing data of lowercase/uppercase/name. Eg:

"iota: ___ α: ___ B: ___ ." You fill in: ι I A *alpha* β *beta*.

Ω: ________ Υ: ________ H: ________

σ: ________ γ: ________ ξ: ________

lambda ________ rho ________ delta ________ mu ________

P3: Mon. 11 Sep Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}} =$ ________ $+\mathbf{i}\cdot$ [________].

Thus $\mathrm{Im}\left(\dfrac{5-\mathbf{i}}{2+3\mathbf{i}}\right) =$ ________ .

By the way, $|5-3\mathbf{i}| =$ ________ .

P4: Wed. 13 Sep Write the free vars in each of these expressions.

$$\overbrace{\exists n \in \mathbb{N}\colon\ f(n) \subset \underbrace{\bigcup_{\ell=p-4}^{p+7} \underbrace{\left\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\right\}}_{E1}}_{E2}}^{E3}$$

$E3$: ________ . $E2$: ________ . $E1$: ________ .

P5: Fri. 15 Sep The number of permutations of "PREPPER", as a multinomial coefficient, is ______ $\overset{\text{numeral}}{\equiv}$ ______.

P6: Mon. 18 Sep

The number of ways of picking 4 objects from 8 types is $\left[\!\left[{8 \atop 4} \right]\!\right] \underset{\text{coeff}}{\overset{\text{Binom}}{\equiv}} \Big($ ______ $\Big) \underset{\text{numeral}}{\overset{\text{Integer}}{\equiv}}$ ______.

And

$\left[\!\left[{8 \atop 4} \right]\!\right] = \left[\!\left[{T \atop K} \right]\!\right]$, where $T=$ ______ $\neq 8$, and $K=$ ______ $\neq 4,1$.

P7: Fri. 22 Sep Write the truth-table for $B \Rightarrow [[\neg B] \Rightarrow C]$.

B	C	$\neg B$	$[\neg B] \Rightarrow C$	$B \Rightarrow [[\neg B] \Rightarrow C]$
F	F			
F	T			
T	F			
T	T			

P8: Fri. 22 Sep LBolt gives $G := \text{GCD}(413, 294) =$ ______. And $413S + 294T = G$, where $S=$ ______ & $T=$ ______ are integers.

Continued on next page...

Monday, 25Sep. is YOM KIPPUR, no MQ; if you miss class, get notes from several colleagues. Mr. Aaron Thrasher will teach.

Wednesday, 27Sep.: HOME A due, BOC!, wATMP! The write-up is *carefully* *stapled*, pages numbered from 1 to N, with page 1 being the Problem Sheet that you printed out,

Friday, 29Sep. is CLASS A, the part of exam-A that students take individually. (Your score is the sum of HOME A with your CLASS A). This is a closed-book, open-brain exam. It is helpful to bring both lined and unlined paper. Bring several writing implements. Make sure they write **Dark**.

No *faint pencil* ***!***

P9: Wed. 27 Sep Compute the real $\alpha =$ .... such that

$$*: \qquad 3^{\alpha} \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \quad = \quad \sum_{j=0}^{1995} \binom{1995}{j} 8^j .$$

[*Hint:* The Binomial Theorem]

PA: Wed. 11 Oct For $K \in [5 \,..\, \infty)$, a closed formula for $\displaystyle\sum_{n=3}^{K} \frac{1}{n\cdot[n+2]}$ is ⌞. . . . .⌟.

[*Hint:* What an *Astronomical* problem!]

PB: Fri. 13 Oct For $K \in [10 \,..\, \infty)$, a closed formula for $\displaystyle\sum_{n=4}^{K} \frac{1}{n\cdot[n+2]}$ is ⌞. . . . .⌟.

[*Hint:* What an *Astronomical* problem!]

PC: Wed. 20 Oct Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.

Define $\omega\,\mathbf{U}\,\lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].

Define $\omega\,\mathbf{I}\,\lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.

So "**U** *is an equiv-relation*" is: *T* *F*

So "**I** *is an equiv-relation*" is: *T* *F*

Mon. 23 Oct *Home-B due, BoC, wATMP*

Wednesday, 25Oct2023 is CLASS B, the part of exam-B that students take individually. This is a closed-book, open-brain exam. It is helpful to bring both lined and unlined paper. Bring several writing implements. Make sure they write **Dark**.

No *faint pencil* ***!***

PD: Fri. 27 Oct On a 4-set, there are ________ many equivalence relations.

PE: Mon. 20 Nov The number of students in class today (including me) is ________.

PF: Mon. 27 Nov Map $f(k,n) \coloneqq 2^k \cdot [1+2n]$ is a bijection from $\mathbb{N}\times\mathbb{N} \hookrightarrow\!\!\!\!\rightarrow \mathbb{Z}_+$. And $f^{-1}(176) = (\_\_\_\_, \_\_\_\_)$.

PH Ungiven:. Let $\mathbb{S}_{\mathbb{N}}$ be the set of *permutations of* $\mathbb{N}$. Circle those of following sets which are equinumerous with $\mathbb{N}^{\mathbb{N}}$:

$\mathbb{N}$ $\mathbb{R}$ $\mathbb{N}\times\mathbb{R}$ $2^{\mathbb{R}}$ $\mathbb{R}^{\mathbb{N}}$ $\mathbb{R}^{\mathbb{R}}$ $\mathbb{S}_{\mathbb{N}}$

[Schröder-Bernstein is useful for some of these.]

Games Party: !! Wed. 06 Dec *Bring games and look photogenic, for our traditional* Games Party, from 10:40 am to 4:40 pm, *at Pascal's Cafe.*