Plex
MAA4402

Quizzes Q

Thursday 18Jan2024

Number Sets. Expression $k \in \mathbb{N}$ [read as "k is an element of $\mathbb{N}$" or "k in $\mathbb{N}$"] means that k is a natural number; a ***natnum***. Expression $\mathbb{N} \ni k$ [read as "$\mathbb{N}$ ***owns*** k"] is a synonym for $k \in \mathbb{N}$.

$\mathbb{N}$ = natural numbers = $\{0, 1, 2, \dots\}$.

$\mathbb{Z}$ = integers = $\{\dots, -2, -1, 0, 1, \dots\}$. For the set $\{1, 2, 3, \dots\}$ of positive integers, the ***posints***, use $\mathbb{Z}_+$. Use $\mathbb{Z}_-$ for the negative integers, the ***negints***.

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid p \in \mathbb{Z} \text{ and } q \in \mathbb{Z}_+\}$. Use $\mathbb{Q}_+$ for the positive rationals and $\mathbb{Q}_-$ for the negative rationals.

$\mathbb{R}$ = reals. The ***posreals*** $\mathbb{R}_+$ and the ***negreals*** $\mathbb{R}_-$.

$\mathbb{C}$ = complex numbers, also called the ***complexes***.

For $\omega \in \mathbb{C}$, let "$\omega > 5$" mean "ω *is* ***real*** <u>and</u> $\omega > 5$". [Use the same convention for $\geq, <, \leq$, and also if 5 is replaced by any real number.]

Use $\overline{\mathbb{R}} = [-\infty, +\infty] := \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the ***extended reals***.

An "***interval of integers***" $[b \,..\, c)$ means the intersection $[b, c) \cap \mathbb{Z}$; ditto for open and closed intervals. So $[e \,..\, 2\pi] = \{3, 4, 5, 6\} = [3 \,..\, 6] = (2 \,..\, 6]$. We allow b and c to be $\pm\infty$; so $(-\infty \,..\, -1]$ is $\mathbb{Z}_-$. And $[-\infty \,..\, -1]$, is $\{-\infty\} \cup \mathbb{Z}_-$.

Floor function: $\lfloor \pi \rfloor = 3$, $\lfloor -\pi \rfloor = -4$. Ceiling fnc: $\lceil \pi \rceil = 4$. Absolute value: $|-6| = 6 = |6|$ and $|-5 + 2i| = \sqrt{29}$.

Mathematical objects. Seq: '*sequence*'. poly(s): '*polynomial(s)*'. irred: '*irreducible*'. Coeff: '*coefficient*' and var(s): '*variable(s)*' and parm(s): '*parameter(s)*'. Expr.: '*expression*'. Fnc: '*function*' (so ratfnc: means rational function, a ratio of polynomials). trnfn: '*transformation*'. cty: '*continuity*'. cts: '*continuous*'. diff'able: '*differentiable*'. CoV: '*Change-of-Variable*'. CoI: '*Constant of Integration*'. LoI: '*Limit(s) of Integration*'. RoC: '*Radius of Convergence*'.

Soln: '*Solution*'. Thm: '*Theorem*'. Prop'n: '*Proposition*'. CEX: '*Counterexample*'. eqn: '*equation*'. RhS: '*RightHand side*' of an eqn or inequality. LhS: '*lefthand side*'. Sqrt or Sqroot: '*square-root*', e.g, "the sqroot of 16 is 4". Ptn: '*partition*', <u>*but*</u> pt: '*point*' as in "a fixed-pt of a map".

Binop: '*Binary operator*'. Binrel: '*Binary relation*'.

FTC: '*Fund. Thm of Calculus*'. IVT: '*intermediate-Value Thm*'. MVT: '*Mean-Value Thm*'.

The ***logarithm*** function, defined for $x>0$, is $\log(x) := \int_1^x \frac{dv}{v}$. Its inverse-fnc is $\exp()$.

For $x>0$, then, $\exp(\log(x)) = x = e^{\log(x)}$. For real t, naturally, $\log(\exp(t)) = t = \log(e^t)$.

PolyExp: '*Polynomial-times-exponential*', e.g, $[3 + t^2] \cdot e^{4t}$. PolyExp-sum: '*Sum of polyexps*'. E.g, $f(t) := 3te^{2t} + [t^2] \cdot e^t$ is a polyexp-sum.

Phrases. WLOG: '*Without loss of generality*'. IFF: '*if and only if*'. TFAE: '*The following are equivalent*'. ITOf: '*In Terms Of*'. OTForm: '*of the form*'. FTSOC: '*For the sake of contradiction*'. And ※="Contradiction".

IST: '*It Suffices To*', as in ISTShow, ISTExhibit.

Use w.r.t: '*with respect to*' and s.t: '*such that*'.

Latin: e.g: *exempli gratia*, '*for example*'. i.e: *id est*, '*that is*'. N.B: *Nota bene*, '*Note well*'. inter alia: '*among other things*'. QED: *quod erat demonstrandum*, meaning "end of proof".

Sequence notation. A *sequence* $\vec{x}$ abbreviates $(x_0, x_1, x_2, x_3, \dots)$. For a set Ω, expression "$\vec{x} \subset \Omega$" means $[\forall n\colon x_n \in \Omega]$. Use $\text{Tail}_N(\vec{x})$ for the subsequence

$$(x_N, x_{N+1}, x_{N+2}, \dots)$$

of $\vec{x}$. Given a fnc $f\colon\Omega\to\Lambda$ and an Ω-sequence $\vec{x}$, let $f(\vec{x})$ be the Λ-sequence $(f(x_1), f(x_2), f(x_2), \dots)$.

Suppose Ω has an addition and multiplication. For Ω-seqs $\vec{x}$ and $\vec{y}$, then, let $\vec{x} + \vec{y}$ be the sequence whose n^{th} member is $x_n + y_n$. I.e

$$\vec{x} + \vec{y} \quad = \quad [n \mapsto [x_n + y_n]] \,.$$

Similarly, $\vec{x} \cdot \vec{y}$ denotes seq $[n \mapsto [x_n \cdot y_n]]$.

Plex notation. Let *SCC* mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{C}$ be the (open) region C encloses, and let $\widehat{C}$ mean C together with $\mathring{C}$. So $\widehat{C}$ is $C \sqcup \mathring{C}$; it is automatically simply-connected and is a closed bounded set.

Plex quizzes during Add/Drop. *These do <u>not</u> count for a grade.*

DNC 1: Fri. 12 Jan Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}} = \underline{\hspace{2cm}} + \mathbf{i}\cdot\left[\underline{\hspace{2cm}}\right]$.

Thus $\dfrac{5-\mathbf{i}}{2+3\mathbf{i}} = \underline{\hspace{4cm}} + \mathbf{i}\cdot\left[\underline{\hspace{4cm}}\right]$.

By the way, $|5-3\mathbf{i}| = \underline{\hspace{4cm}}$.

Plex [2024g] quizzes so far...

(The lowest MQ score is dropped. In consequence, there is *no make-up* for the first missed MQ.)

[Monday, 15Jan2024: MLK Day, *No class*]

Q1: Wed. 17 Jan *Reals* $x=$ ______ and $y=$ ______ where $x+\mathbf{i}y = [1+\mathbf{i}]^{86}$. [*Hint: Multiplying complexes multiplies their moduli, and adds their angles.*]

Q2: Fri. 19 Jan Number $\left[\mathbf{i}+\sqrt{3}\right]^{70} = x+\mathbf{i}y$, for real numbers $x=$ ______ and $y=$ ______.
[*Multiplying complexes multiplies their moduli* (*absolute-values*), *and adds their angles.*]

Q3: Mon. 22 Jan With $v := \exp(-2+5\mathbf{i})$, then $|v| =$ ______.
This $|v|$ lies in [circle the correct interval]
$[0,\frac{1}{2})$, $[\frac{1}{2},1)$, $[1,2)$, $[2,4)$, $[4,8)$, $[8,\infty)$.

Q4: Wed. 24 Jan Number $6\cdot\exp\left(\mathbf{i}\cdot\frac{5\pi}{3}\right)$ equals $x + y\mathbf{i}$ for reals
$x=$ ______ and $y=$ ______.

Q5: Fri. 26 Jan *The empty-set is path-connected:* T F
Punctured ball $\mathrm{PBal}_2(3\mathbf{i})$ *is path-connected:* T F
$\mathrm{Sph}_2(5\mathbf{i}) \cap \mathrm{Sph}_2(\mathbf{i})$ *is path-connected:* T F
$\mathrm{Sph}_2(4\mathbf{i}) \cup \mathrm{Sph}_2(-\mathbf{i})$ *is path-connected:* T F
$\mathrm{Sph}_2(5\mathbf{i}) \cup \mathrm{CldBal}_2(\mathbf{i})$ *is closed:* T F

Q6: Fri. 09 Feb Cross-ratio $[z,\ 2+\mathbf{i},\ 4\mathbf{i},\ 3] = \frac{\mathtt{a}z+\mathtt{b}}{\mathtt{c}z+\mathtt{d}}$, where
a= ______, b= ______,
c= ______, d= ______.

Q7: Wed. 08 Mar Let $f(z) := \mathrm{e}^{4z}/\sin(3z)$.
For $n\in\mathbb{Z}$, then, $\mathrm{Res}(f, n\pi) =$ ______.
And $\mathrm{Res}(f, \frac{\pi}{2}) =$ ______.

Q8: Mon. 18 Mar *Complex Analysis is Fun!* [Circle]: **Yes** **True**
PS $\cos(z)=$ ______ $+\dots$
With $f(z) := z/[\cos(z)-1]$, let $R_k := \mathrm{Res}(f, 2\pi k)$.
Then $R_0 =$ ______ and $R_3 =$ ______.
[*Hint:* Periodicity $\cos(z+2\pi k) = \cos(z)$.]

Bonus-T: Wed. 03 Apr Let $f(z) := \frac{z^3}{z^2-9}$.
Then $\mathrm{Res}(f,\ 3) =$ ______.