PRACTICE: Binomials, $\mathbb{C}$-arithmetic

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Welcome. [This is not a proctice exam; just some proctice problems.] Note: ITO=In-Terms-Of.

P1:

a Values $[\![\pi \uparrow 3]\!]=$ ⌞.....⌟ and $[\![\frac{5}{3} \downarrow 4]\!]=$ ⌞.....⌟.

b Binomial coefficient $\binom{7}{4}=$ ⌞.....⌟ $=$ ⌞.....⌟.

c Multinomial coefficient $\binom{9}{4,2,3}=$ ⌞.....⌟ $=$ ⌞.....⌟.

[*Note:* Write your ans. ITOf factorials, then **also** write it as a single integer, or product of two, **without** factorials.]

d In $\left[5x^2+4y+z^3+7\right]^{20}$, compute these coeffs:

$\text{Coeff}(x^6z^8) =$ ⌞.....⌟.

$\text{Coeff}(y^5z^6) =$ ⌞.....⌟

[You may write answers as a product numbers, powers and multinomial-coeffs.]

P2:

e Blanks$\in\mathbb{R}$. So $\frac{1}{2+3\mathbf{i}}=$ ⌞.....⌟ $+\mathbf{i}\cdot$ ⌞.....⌟.

Thus $\text{Im}\left(\frac{5-\mathbf{i}}{2+3\mathbf{i}}\right) =$ ⌞.....⌟.

By the way, $|5-3\mathbf{i}| =$ ⌞.....⌟.

f Let $A := 2+\mathbf{i}$, $B := 1-\mathbf{i}$, $P := 5$, $Q := 3\mathbf{i}$, $\boldsymbol{\alpha} := 1$ and $\boldsymbol{\beta} := 1+3\mathbf{i}$. Complex numbers w and z satisfy

$$\begin{aligned} Aw + Bz &= \boldsymbol{\alpha} \quad \textit{and} \\ Pw + Qz &= \boldsymbol{\beta}. \end{aligned}$$

Then $w =$ ⌞.....⌟.

g *Reals* $x=$ ⌞.....⌟ and $y=$ ⌞.....⌟ where $x+\mathbf{i}y = [1+\mathbf{i}]^{86}$. [*Hint: Multiplying complexes multiplies their moduli, and adds their angles.*]

h Complex number $[x + \mathbf{i}y]^2 = -8\,\mathbf{i}$, for *real numbers* $x>y$, where $x=$ ⌞.....⌟ and $y=$ ⌞.....⌟.

For students who have seen the complex-exponential function:

i Number $6\cdot\exp\left(\mathbf{i}\cdot\frac{5\boldsymbol{\pi}}{3}\right)$ equals $x + y\mathbf{i}$ for reals $x=$ ⌞.....⌟ and $y=$ ⌞.....⌟.

With $v := \exp(-2+5\mathbf{i})$, then $|v| =$ ⌞.....⌟.

And $|v|$ lies in (circle the correct interval)

$$[0,\tfrac{1}{2}),\quad [\tfrac{1}{2},1),\quad [1,2),\quad [2,4),\quad [4,8),\quad [8,\infty).$$