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Quizzes Q

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In $\mathbb{C}^4$, consider $\mathbf{g}_1$, $\mathbf{g}_2$, $\mathbf{g}_3$, column-vectors

$$\begin{bmatrix}1\\ \mathbf{i}\\ 0\\ 0\end{bmatrix}, \quad \begin{bmatrix}0\\ 2\\ \mathbf{i}\\ 0\end{bmatrix}, \quad \begin{bmatrix}0\\ 6\mathbf{i}\\ 0\\ 6+6\mathbf{i}\end{bmatrix},$$

respectively. With IP = DotProduct, compute pairwise orthogonal $\mathbf{b}_1 := \mathbf{g}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$ using Gram-Schmidt, yielding

$\mathbf{b}_2 = [\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_]^{\mathrm{t}}$ *and*

$\mathbf{b}_3 = [\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_]^{\mathrm{t}}$.

Soln. Using our formula for ortho-projection:

```
(ip-select t)  =>  "Using COLVEC-inner-product."
(use-ring GaussRational-ring)
(defaliasqq mmc mat-make-colvec)
(setq g1 (mmc 1    imi    0       0  )
      g2 (mmc 0     2    imi      0  )
      g3 (mmc 0 (* 6 imi) 0   #C(6 6)) )
  [ 1 ]   [ 0 ]   [ 0       ]
  [ i ]   [ 2 ]   [      6i ]
  [ 0 ] , [ i ] , [ 0       ]
  [ 0 ]   [ 0 ]   [ 6 + 6i ]

(setq B1 G1) ;; Computing B1:

;; Computing B2:
(setq ratio (/ (ip B1 G2) (ip B1 B1))) => -i    [ -i ]
(setq ProjG2onB1 (mat-scal-mult ratio B1)) =>   [  1 ]
                                                [  0 ]
                                        [ i ]   [  0 ]
                                        [ 1 ]
(setq B2 (mat-sub G2 ProjG2onB1))  =>   [ i ]
                                        [ 0 ]

(setq ratio (/ (ip B1 G3) (ip B1 B1))) => 3     [  3 ]
(setq ProjG3onB1 (mat-scal-mult ratio B1)) =>   [ 3i ]
                                                [  0 ]
                                                [  0 ]

(setq ratio (/ (ip B2 G3) (ip B2 B2)))  => 2i   [ -2 ]
(setq ProjG3onB2 (mat-scal-mult ratio B2)) =>   [ 2i ]
                                                [ -2 ]
                                                [  0 ]

(setq B3 (mat-sub G3 (mat-add ProjG3onB1 ProjG3onB2)))
    [   -1 ]
    [    i ]
    [    2 ]
    [ 6+6i ]
```

♦

Ans. So $\mathbf{b}_2 = [\mathbf{i}, 1, \mathbf{i}, 0]^{\mathrm{t}}$ *and* $\mathbf{b}_3 = [-1, \mathbf{i}, 2, 6+6\mathbf{i}]^{\mathrm{t}}$.

We test whether the **b**-tuple is pairwise-ortho:

```
(list (ip b1 b2) (ip b1 b3) (ip b2 b3))  =>  (0 0 0)


;;;;;;;;;;;;;;;;
;; Checking that Span(B1, B2, B3) = Span(G1, G2, G3).
;;;;;;;;;;;;;;;;
(setq Bert (mat-Horiz-concat B1 B2 B3   G1 G2 G3))
    [  1  i    -1     1  0     0  ]
    [  i  1     i     i  2    6i  ]
    [  0  i     2     0  i     0  ]
    [  0  0  6+6i     0  0  6+6i  ]


(rref-mtab-beforecol Bert)
  JK: Found 3 pivots before the sixth column.

    c0  c1  c2   c3  c4  c5    Row operations
   |--------------------|
r0 | 1   0   0    1  -i   3 |  1/2       -[1/2]i    0       0
r1 | 0   1   0    0   1  2i |  -[1/3]i    1/3     -[1/3]i   0
r2 | 0   0   1    0   0   1 |  -1/6      -[1/6]i    1/3      0
r3 | 0   0   0    0   0   0 |  1+i       -1+i      -2-2i    1
   |--------------------|
```

The *RREF* shows that, were we to exchange columns c_4, c_5, c_6 with c_1, c_2, c_3, we would have three pivits in c_4, c_5, c_6. Consequently $\mathrm{Spn}(c_4, c_5, c_6) = \mathrm{Spn}(c_1, c_2, c_3)$. ♦