Complex Analysis **Exam Gamma** Prof. J. King

1999 April 26 5:30PM-7:30PM LIT203

Notes. The symbol $C_R(Q)$ denotes the circle of radius R, centered at $Q \in \mathbb{C}$.

Please be sure to write expressions unambiguously e.g, the expression "$1/a+b$" should be parenthesized either $[1/a]+b$ or $1/[a+b]$ so. Similarly, write "$\sin(x)$" rather than "$\sin x$". Be careful with negative signs!

γ1: [50 × 4 = 200 points]. Quickos: Fill in the blanks below, writing your answer in simplest form unless otherwise indicated. **Show no work.** There is no (well...little) partial credit, so carefully check that you have written what *you* mean.

(a) Compute the radius-of-convergence of pow.series

$$\sum_{n=0}^{\infty} \frac{2^n + 3^n}{1+5n} z^n . \qquad \text{RoC}= \underline{\qquad\qquad\qquad} .$$

(b) Let $R \in [0, +\infty]$ be the radius-of-convergence of a power series $\sum_{n=0}^{\infty} b_n z^n$. Let $a_n := |b_n|$. Give a formula for R in terms of the a_n numbers and limsup and/or liminf and/or lim.

$$R = \underline{\qquad\qquad\qquad} .$$

(c) Please compute the first four coefficients of the power series, centered at zero, of $1/\exp$.

$$\frac{1}{e^z} \;=\; \underline{\quad} + \underline{\quad}\, z + \underline{\quad}\, z^2 + \underline{\quad}\, z^3 + \ldots .$$

(d) Let $g(z) := \dfrac{1}{z \cdot [\sin(z)]^2}$. Compute the residue at zero. $\operatorname{Res}_g(0) = \underline{\qquad\qquad}$.

Notes. For the next two questions you should show all work on separate sheets of paper that you provide. Please staple those sheets to this Problem-sheet, with the Problem-sheet as the first page. Please put your name on **each page** of your write-up, and number the pages consecutively, including diagram pages. Carefully and cogently write up your solutions in complete English sentences (*please write only on every third line, so that I have room for comments*) drawing good large pictures, where appropriate. If you use a theorem, please **state** it precisely, and overtly check that its hypotheses are satisfied in your application.

γ2: [220 points]. For an $R > 0$ and $Q \in \mathbb{C}$, suppose that h is analytic on $\operatorname{Disk}_R(Q)$. For $K = 0, 1, 2, \ldots$, define the K-th Taylor polynomial

$$\operatorname{Tay}_K(w) := \sum_{n=0}^{K-1} \frac{h^{(n)}(Q)}{n!} \cdot \left[w - Q\right]^n .$$

For each $w \in \operatorname{Ball}_R(Q)$, define $E_K(w)$ by

$$h(w) \;=\; \operatorname{Tay}_K(w) \;+\; \left[w - Q\right]^K \cdot E_K(w) ,$$

and $E_K(Q) := 0$.

(i) State the integral-form of E_K, by **filling in** the missing integrand.

$$E_K(w) = \frac{1}{2\pi i} \int_{C_R(Q)} \underline{\qquad\qquad\qquad\qquad\qquad} \, dz .$$

(ii) For notational simplicity, henceforth assume that $Q = 0$. Use the Cauchy Integral Formula Theorem to *prove* your form above.

(iii) For each $w \in \operatorname{Ball}_R(Q)$, prove that

$$\lim_{K \to \infty} w^K \cdot E_K(w) \;=\; 0 .$$

γ3: [200 points]. Use the residue calculus to compute

$$I := \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{5z+1}{[z^4+4][z^2+3]} \, dz .$$

To save computation, you may define some **explicit** complex numbers $P_1, \ldots, P_L$ (you decide what L is) and **explicit** functions $f_1, \ldots, f_L$, and then may express your answer in the form

$$I \;=\; f_1(P_1) + \cdots + f_L(P_L) .$$

You do not need to actually perform the addition.

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