Staple!

Ord: ___ ___

Sets and Logic MHF3202 7860 | **Practice-P** | Prof. JLF King *Whenever...*

P1: Short answer. Show no work. Write LARGE.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

a Which action *loses* pts? (circle)

Writing-in-sentences

Writing-t-different-from-$+$

Tiny - tiny - writing

Writing-LARGE

b On $\Omega := [1..19]\times[1..19]$, define binary-relation $\mathbf{S}$ by: $(x,\alpha)\,\mathbf{S}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{20} y\cdot\alpha$. Statement "*Relation* $\mathbf{S}$ *is an* ***equivalence relation***" is: T F
The answer is FALSE. *Produce* three specific pairs p,q,r such that $p\,\mathbf{S}\,q$ and $q\,\mathbf{S}\,r$, *yet* $p \not\mathbf{S}\, r$.

c Note that $\text{GCD}(15,21,35)=1$. Find particular integers S,T,U so that $15S+21T+35U=1$:
$S=$ ______, $T=$ ______, $U=$ ______.
[*Hint:* $\text{GCD}\big(\text{GCD}(15,21),35\big)=1$.]

d On a 5-set, let K be the number of partitions which use *only* atom sizes 1 and 2. [A partition need not use both sizes.] Then $K=$ ______. [Multinomial coefficients are useful.]

e Let N be the number of permutations of the letters in ABRACADABRA. As a multinomial-coeff, $N = \binom{\qquad}{\qquad}$. [Write the bottom integers in **increasing** order, $p_1 \le p_2 \le \dots$. The bottom integers should sum to the top integer.] Written as product-of-binomials, $N =$ ______.
Evaluate each binomial as an integer, and write N as a product of these integers: $N =$ ______.

f For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) := \Big\{k \in [1..N] \;\Big|\; \exists d \in \mathcal{S} \text{ with } d \bullet\!| k\Big\}.$$

For $\mathcal{S} := \{6,7,10\}$, the Inclusion-Exclusion formula (using the floor fnc) for the number of elements, $|\boldsymbol{\mu}_{\mathcal{S}}(N)|$, is

______

(Write your answer, using the floor function as appropriate, in form [term + term + term] $-$ [term + term + term] $+$ term.
The terms are computed from the $\{6,7,10\}$ numbers.)

When $N := 67$, then, $|\mu_{\{6,7,10\}}(67)| =$ ______.

g Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\boldsymbol{\Omega} := \mathbb{N}$, consider injections $g\colon \mathbf{X}\hookrightarrow\boldsymbol{\Omega}$ and $h\colon\boldsymbol{\Omega}\hookrightarrow\mathbf{X}$, defined by

$$g(x) := 3x \quad\text{and}\quad h(y) := y+5\,.$$

Schröder-Bernstein produces a set $B \subset h(\boldsymbol{\Omega}) \subset \mathbf{X}$ st., letting $F := \mathbf{X} \smallsetminus B$, the fnc $\theta\colon \mathbf{X} \twoheadrightarrow \boldsymbol{\Omega}$ is a *bijection*, where

$$*: \qquad \theta\restriction_F := g\restriction_F \quad\text{and}\quad \theta\restriction_B := h^{-1}\restriction_B\,.$$

For this (g,h), the (F,B) pair is unique. Computing, $\theta(\mathbf{56})=$ ______. $\theta(\mathbf{137})=$ ______. $\theta^{-1}(\mathbf{603})=$ ______.

h Let $T(m) := \frac{1}{2}m[m+1]$ be the m^{th} triangular number. An explicit bijection $F\colon \mathbb{N}\times\mathbb{N} \hookrightarrow\!\!\!\!\rightarrow [-4..\infty)$ is
$F(n,k) :=$ ______.

Continued...

OYOP: *In grammatical English* ***sentences****, write each essay on every* 2^{nd} *line* (usually)*, so that I can easily write between the lines.*

P2: Carefully STATE the Cardinality Exponentiation Lemma [CE-Lemma].

Prove the CE-Lemma.

P3: Let $\mathcal{P}_\infty$ denote the collection of all infinite subsets of $\mathbb{N}$. Define a relation $\doteqdot$ on $\mathcal{P}_\infty$ by: $A \doteqdot B$ IFF $A \cap B$ is infinite.

Either prove that $\doteqdot$ *is* transitive, or else produce three explicit sets A,B,$C \in \mathcal{P}_\infty$ showing that $\doteqdot$ is *not* transitive.

P1:	___ ___ ___	000pts
P2:	___ ___ ___	000pts
P3:	___ ___ ___	000pts
Total:	___	0pts

NAME: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....