Staple!

Team B____

Sets and Logic
MHF3202 7860

Home-B

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Wednesday, 06Mar2024

Due **BoC, Wedn, 20Mar2024**, wATMP! **Print this problem-sheet**; it is the first page of your write-up, with the blanks filled in (handwritten). Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. [Put ordinal, Team-# and sign Honor Code.]

B1: *Show no work.*

____ a A K-set Ω has .... **non-symmetric** binrels. Its number of **anti-symmetric** binrels is ....

[**Note:** Do not confuse symmetric with reflexive. Be *careful* on this problem.]

____ b Sum $\binom{45}{19} + 2\binom{45}{20} + \binom{45}{21} = \binom{T}{B}$, where $T =$ ...... and $B =$ .... Using the same idea,

$$\binom{32}{13} + 3\binom{32}{14} + 3\binom{32}{15} + \binom{32}{16} = \binom{\tau}{\beta},$$

where $\tau =$ .... and $\beta =$ ....

____ c On $\Omega := [1..29]\times[1..29]$, define binary-relation $\mathbf{C}$ by: $(x,\alpha)\ \mathbf{C}\ (y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* $\mathbf{C}$ *is an* ***equivalence relation***" is: *T* *F*

____ d Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\mathbf{\Omega}$, both strict. Define binrel $\ll$ on $\mathcal{S}\times\mathbf{\Omega}$ by:

$$(b,\beta) \ll (c,\gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:
Relation $\ll$ *is a total-order.* *T* *F*

Suppose $\prec$ *and* $<$ *are each well-orders.*

Then $\ll$ *is a well-order.* *T* *F*

Carefully TYPE *your two essays,* ***double–spaced.*** *I suggest* LaTeX.

B2: Recall *Rabbits and Lights* from the zoomester's beginning: To your right are lights $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \ldots$. Each light has a toggle button; Press&release: the light illuminates; P&R again, it is extinguished.

Off to your left is a queue of rabbits; so we have

$$\ldots \mathcal{R}_3\ \mathcal{R}_2\ \mathcal{R}_1\ \ \mathcal{L}_1\ \mathcal{L}_2\ \mathcal{L}_3\ \mathcal{L}_4, \ldots$$

All the lights are initially off. *If* rabbit-α (i.e, $\mathcal{R}_\alpha$) jumps, then he will hop on lights $\mathcal{L}_\alpha, \mathcal{L}_{2\alpha}, \mathcal{L}_{3\alpha}, \ldots$, turning them all on. If rabbit-β now jumps, he will change the state of lights β, 2β, $3\beta, \ldots$, turning some on, and some off.

A Map f. A (finite or infinite) set $\mathsf{R} = \{\alpha_1, \alpha_2, \ldots\}$ of rabbit-indices is an element of powerset $\boldsymbol{\mathcal{P}} := \mathcal{P}(\mathbb{Z}_+)$. After those rabbits jump, we have a (finite or infinite) set $\mathsf{L} = \{\beta_1, \beta_2, \beta_3, \ldots\}$ of indices of illuminated lights. Define $f{:}\boldsymbol{\mathcal{P}}\to\boldsymbol{\mathcal{P}}$ by $f(\mathsf{R}) := \mathsf{L}$.

Our first-day class showed [*involution argument, and re-argued using the divisor-count* $\boldsymbol{\tau}$*-fnc*] that $f(\mathbb{Z}_+)$ is the set $\{1,4,9,\ldots\}$ of squares. Evidently $f(\varnothing) = \varnothing$ and $f(\{1,2\}) = \text{Odds}$. □

Q1 For each of the following questions, produce *either* a CEX [counterexample] *or* a formal proof.
Is f injective? Is f surjective?

Q2 For $\mathsf{L} \in \text{Range}(f)$, give an algorithm to produce an R for which $f(\mathsf{R}) = \mathsf{L}$. If you program, can you implement your algorithm in computer code?

Q3 Produce a (*non-trivial*) commutative, associative binop $\$: \boldsymbol{\mathcal{P}}\times\boldsymbol{\mathcal{P}} \to \boldsymbol{\mathcal{P}}$ which satisfies

$$\forall \mathsf{R},\mathsf{R}': \quad f(\mathsf{R} \$ \mathsf{R}') = f(\mathsf{R}) \$ f(\mathsf{R}').$$

What can you tell me about this binary operator?

Q4 What are all the f–fixed-points; those rabbit-lists R with $f(\mathsf{R}) = \mathsf{R}$?

What can you say about the dynamics of f? —does it have periodic points of order 2? 3? ...?

What is $f(f(\mathbb{Z}_+)) \overset{\text{note}}{=\!=} f(\text{Squares})$? (Conjecture? Computer simulation?)

B3: The ***Threeish-numbers*** comprise $\mathcal{T} := 1 + 3\mathbb{N}$. In terms of PoP-factorization $\mathsf{T} = p_1^{E_1} \cdot \ldots \cdot p_K^{E_K}$ [where $p_1 < \ldots < p_K$ are $\mathbb{Z}$-primes, and each E_j is a posint]:

i Give/prove an IFF-characterization for when $\mathsf{T} \in \mathcal{T}$.

ii Give/prove an IFF-char. of when T is Threeish-*irreducible*.

iii Give/prove an IFF-char. of when T is Threeish-*prime*.

iv Using theorem(s) from The Web, prove or disprove: *"There are* ∞*ly many Threeish-primes."*

B1:	___ ___ ___	135pts
B2:	___ ___ ___	100pts
B3:	___ ___	85pts
Total:	___ ___ ___	320pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord:
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