Staple!

Team B____

Sets and Logic
MHF3202 139A

Home-B

Prof. JLF King
Tues., 18Oct2022

Due **BoC, Monday, 24Oct2022**, wATMP! **Print this problem-sheet**; it is the first page of your write-up, with the blanks filled in (handwritten). Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$. [Put ordinal, Team-# and sign HONOR CODE.]

B1: *Show no work.*

___ ___ a Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}\colon\ f(n) \subset \overbrace{\underbrace{\bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \ \equiv_5\ x^2\}}_{E1}}_{E2}}^{E3}$$

$E3$: .... $E2$: .... $E1$: ....

___ ___ b Let N be the number of permutations of the letters in ABRACADABRA. As a multinomial-coeff, $N = \binom{\qquad}{\qquad}$. [Write the bottom integers in **increasing** order, $p_1 \leq p_2 \leq \ldots$. The bottom integers should sum to the top integer.] Written as product-of-binomials, $N =$ ....

Evaluate each binomial as an integer, and write N as a product of these integers: $N =$ ....

___ ___ c The number, U, of permUtations of $[1..6]$ which have *neither* $(4\ 1)$ *nor* $(6\ 3\ 2)$ is ....

[Write in Incl-Excl form: $\square - \square + \square - \square + \ldots$ as appropriate.]

As a single numeral, $U =$ ....

___ ___ d Let $\mathcal{P}_\infty$ denote the family of all ***co-finite*** subsets of $\mathbb{N}$. That is, a subset $S \subset \mathbb{N}$ is an *element* of $\mathcal{P}_\infty$ IFF $\mathbb{N} \setminus S$ is finite. Define relation $\bowtie$ on $\mathcal{P}_\infty$ by: $A \bowtie B$ IFF $A \cap B$ is infinite.

Stmt "*This $\bowtie$ is an equivalence-relation*" is: *T F*

___ ___ e Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\mathbf{\Omega}$, both strict. Define binrel $\ll$ on $\mathcal{S} \times \mathbf{\Omega}$ by:

$$(b, \beta) \ll (c, \gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:
Relation $\ll$ is a total-order. *T F*

Suppose $\prec$ and $<$ are each well-orders.
Then $\ll$ is a well-order. *T F*

Carefully TYPE *your two essays, double–spaced. I suggest* LaTeX, *but other systems are ok too.*

B2: [A dodecahedron is a regular polyhedron having 12 faces, 20 vertices and 30 edges; the faces are pentagons.] Two vertices of a regular dodecahedron are ***cousins*** if they are *distinct* vertices of a common face. [Each vertex has $[3 \cdot 4] - 3 = 9$ cousins.] Write $v \sim w$ to indicate that v and w are cousins. Easily, $\sim$ is symmetric, and anti-reflexive. You can check that $\sim$ is not transitive.

A ***labeling*** of a regular dodecahedron assigns, to each vertex, a *positive integer*. A labeling is ***legal*** IFF *no* pair $v \sim w$ of vertices is assigned the same label.

i Prove there is no legal labeling with vertex-sum [the sum of the 20 labels] equaling 55.

ii Let $\mathcal{S} \subset \mathbb{Z}_+$ be the *set* of vertex-sums obtainable from legal-labelings. Characterize $\mathcal{S}$ explicitly, with proof. You will likely need to construct some particular legal-labelings. [You showed, above, that $\mathcal{S} \not\ni 55$.]

B3: Prove, for each natnum N, that

$$\sum_{k=0}^{N} \left[\binom{N}{k}^2\right] = \binom{2N}{N}.$$

[Can use Double-counting, or Induction.]

B1:	___ ___ ___	135pts
B2:	___ ___	85pts
B3:	___ ___	55pts
Total:	___ ___ ___	275pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....