

Sets and Logic
MHF3202 1079

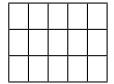
Class-B

Prof. JLF King
Wedn, 25Oct2023**B4:** Short answer. Show no work. Write LARGE.Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.**a** For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course :

Yes

True

Darn tootin'!

20 b Mimicking what we did in class: From the 987×200 game-board, cut-out (remove) the $(35, 150)$ -cell and one other cell at $P = (x, y)$. those choices for P , $(150, 160), (14, 35), (66, 77), (195, 15), (123, 4)$ which, if removed, would leave a board that *definitely can't* be domino-tiled.**15 c** Let \mathcal{P}_∞ denote the family of all *infinite* subsets of \mathbb{N} . Define relation \approx on \mathcal{P}_∞ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt “*This \approx is an equivalence-relation*” is: F**15 d** On a 3-set, there are many equiv.relations.**20 15 e** Let δ_N be the number of derangements of $[1..N]$. Written in Incl-Excl notation (the formula we derived in class), $\delta_{15} =$ Using binom-coeffs and derangements, the number of N -perms with precisely3 fixed-points is: [You many use binom-coeffs and $\delta_1, \delta_2, \dots$ in your answer.]**20 f** A $k \in [1..100]$ is *good* if $k \nmid 2$ or $k \nmid 3$ or $k \nmid 5$. So #Good = . [Hint: Inclusion-exclusion]OYOP: In grammatical English **sentences**, write your essay on every 2nd line (usually), so I can easily write between the lines.**B5:** An **Lmino** (pron. “ell-mino”) comprises three  squares in an “L” shape (all four orientations are allowed). For natnum N , let \mathbf{R}_N denote the $3 \times N$ board: I.e.,  is the \mathbf{R}_5 board. Prove:Theorem: When N is odd, then board \mathbf{R}_N is not Lmino-tilable.You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, labeled pictures.When N is even, our \mathbf{R}_N has exactly many Lmino-tilings.**B4:** _____ 120pts**B5:** _____ 55pts**Total:** _____ 175pts

NAME: _____ Ord: _____

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: _____