

Plex MAA4402 2838 **More-Prac-B** Prof. JLF King Wedn, 08Mar2023

**NB.** For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0$ .

Let **holom** abbreviate “holomorphic”, and **harm.fnc** abbreviate “harmonic function”. Use **PS**=Power Series, **RoC**=Radius-of-Convergence, and **MacSe** for “Maclaurin series”; a **PS** centered at 0. [Below, **ITOf** means “In Terms Of”.]

**B1:** Short answer. Show no work.

**a** The IOP (Individual Optional Project) must be carefully TYPESET. It is due by **2PM** on **Thursday, 27Apr2023**, slid *completely* under my office door, **Little Hall 402** (northeast corner of top floor) Circle: Yes Cool! Thanks

**b** Subset  $K \subset \mathbb{C}$  is *compact* if:  
 .....  
 .....  
 .....  
 .....

**c** On a subset  $G \subset \mathbb{C}$ , consider two cts parametrizations  $\gamma, \sigma: [0, 1] \rightarrow G$  which are loops;  $\gamma(0) = \gamma(1)$  and  $\sigma(0) = \sigma(1)$ . A “*homotopy* in  $G$  from  $\gamma$  to  $\sigma$ ” is  
 .....  
 .....  
 .....  
 .....

**d** Write  $\cos(-2i)$ , which is real, ITOf  $\exp()$  and *finite* add/sub/mul/div:  $\cos(-2i) =$  .....  
 And  $\cos(-2i)$  lies in circle the correct interval  
 $(-\infty, -\frac{1}{5}]$   $(-\frac{1}{5}, \frac{1}{5}]$   $(\frac{1}{5}, 2]$   $(2, 5]$   $(5, 15]$   $(15, 45]$   $(45, \infty)$

**e** Compute  $\int_0^{2\pi} \frac{1}{\cos(\theta) + 6} d\theta =$  .....  
 [Hint: CoV  $z = e^{i\theta}$ .]

OYOP: In *grammatical English sentences*, write your essay on every 2<sup>nd</sup> line (usually), so I can easily write *between the lines*.

**B2:** Below,  $h: \mathbb{C} \rightarrow \mathbb{C}$ , and  $S \subset \mathbb{C}$  is a closed-curve, and  $w \in \mathbb{C}$  is an *appropriate* point.

**$\alpha$**  Detailing the precise conditions needed on  $h$ ,  $S$  and  $w$ , *carefully* state the Cauchy Integral Formula Theorem.

**$\beta$**  Recall the Cauchy Homotopy Thm: Suppose closed-curves  $S$  and  $R$  are homotopic in an open set on which a fnc  $f$  is holomorphic. Then  $\oint_S f = \oint_R f$ .

Use the above CHT to give a formal proof of the Cauchy Integral Formula Theorem. Also draw LARGE pictures showing the ideas in the proof.

NAME: .....

**HONOR CODE:** “I have neither requested nor received help on this exam other than from my professor.”

Signature: .....