Staple!

Ord: ___ ___

Plex MAA4402 2838 | **Practice-B** | Prof. JLF King, Wedn, 08Mar2023

NB. For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

Let **holom** abbreviate "holomorphic", and **harm.fnc** abbreviate "harmonic function". Use PS=Power Series, RoC=Radius-of-Convergence, and MacSe for "Maclaurin series"; a PS centered at 0.

B1: Short answer. Show no work.

___ ___ **a** The IOP (Individual Optional Project) must be carefully TYPESET. It is due by 2PM on Thursday, 27Apr2023, slid *completely* under my office door, Little Hall 402 (northeast corner of top floor) $\boxed{\text{Circle}}$: Yes Cool! Thanks

___ ___ **b** Subset $K \subset \mathbb{C}$ is ***disconnected*** if: ....

....

....

....

___ ___ **c** "Fnc $f:\mathbb{C}\to\mathbb{C}$ is ***continuous*** at $p \in \mathbb{C}$" means [the formal, ε, δ defn]: ....

....

....

....

___ ___ **d** Subset $Q \subset \mathbb{C}$ is ***path-connected*** if: ....

....

....

....

___ ___ **e** Consider $h:G\to\mathbb{C}$, where $G \subset \mathbb{C}$ is not necessarily open. "Fnc h is ***holomorphic*** on G" means:: ....

....

....

....

___ ___ **f** Express this sum as a rational in lowest terms. $\sum_{n=0}^{\infty} \frac{2^n - 5^n}{10^n} =$ ....

g [Below, "Log" is the P.V. of $\mathbb{C}$-logarithm, and $z,w \in \mathbb{C}$.]

If $\mathrm{Re}(z)<0$ then $\mathrm{Log}(z^2) = 2\,\mathrm{Log}(z)$. *AT AF Nei*

If $\mathrm{Re}(z)>1$ then $\mathrm{Log}(z^3) = 3\,\mathrm{Log}(z)$. *AT AF Nei*

If $\mathrm{Re}(z)=0$ and $\mathrm{Im}(z)>2$ then $\mathrm{Log}(\exp(z)) = z$. *AT AF Nei*

If $\mathrm{Re}(z)>0$ and $\mathrm{Re}(w)>0$ then $\mathrm{Log}(z \cdot w) = \mathrm{Log}(z) + \mathrm{Log}(w)$. *AT AF Nei*

So $\mathrm{Log}([1+\mathbf{i}]^{13}) = 13\ln(\sqrt{2}) + y\mathbf{i}$, where $y =$ ....

h The MacSe of $\frac{5}{1-x^3}$ has RoC= ....

$\mathrm{MacSe}(x) =$ ....

Write the first 5 non-zero terms, e.g, $8x^3 + \frac{1}{8}x^6 + \frac{3}{2}x^8 - x^{12} - 7x^{15} + \cdots$.

i $\mathrm{Res}\left(\frac{\mathrm{e}^{2z}}{[z-5]^4}, z{=}5\right) =$ ....

j Compute real integral $J := \int_{-\infty}^{+\infty} \frac{1}{16+x^4}\,\mathrm{d}x$ by converting [as we did in class] to an integral over a D-shaped contour. Our $J =$ ....

k Fnc $\mathsf{u}(x,y) := 2xy + x$ has harmonic conjugate $\mathsf{v}(x,y) =$ ....

OYOP: *In grammatical English* ***sentences****, write your essay on every 2nd line* (usually)*, so I can easily write between the lines.*

B2: Below, $h:\mathbb{C}\to\mathbb{C}$.

-- -- **α** Suppose h is differentiable at the point $3+2\mathbf{i}$. Writing h in real and imaginary parts, $h(x+\mathbf{i}y) = u(x,y) + \mathbf{i}v(x,y)$, *state* the Cauchy-Riemann eqns for h at $3+2\mathbf{i}$.

-- -- **β** Suppose h is differentiable at a point $z \in \mathbb{C}$. Carefully *derive* the Cauchy-Riemann eqns, directly from the defn of "differentiable".