Sets and Logic
MHF3202 2787

Home-A

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Thursday, 20Sep2018

Due **BoC, Monday, 24Sep2018**, wATMP!
Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

A1: *Show no work. Simply fill-in each blank on the problem-sheet.*

a [With $\mathcal{P}()$ the *powerset* operator, let $S :=$ 3-stooges.] Then $|\mathcal{P}(S)| =$ ______ and $|\mathcal{P}(\mathcal{P}(S))| =$ ______.

b Using *only* symbols $\boldsymbol{H}, \boldsymbol{D}, \wedge, \vee, \neg, T, F,], [$, rewrite (in simplest form) expression $[[H \Rightarrow D] \Rightarrow H]$ as ______. Ditto, rewrite $[H \Rightarrow [D \Rightarrow H]]$ as ______.

c $\forall x, z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T F*
$\forall x, z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T F*
For all sets Ω*, there exists a fnc* $f : \mathbb{R} \to \Omega$. *T F*

d The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is ______.
[You may write in form number times multinomial-coeff. You can leave the multinomial-coeff as such, or write ITOf factorials.]

e The number of ways of picking 42 objects from 70 types is $\left[\!\left[\begin{matrix} 70 \\ 42 \end{matrix} \right]\!\right] \stackrel{\text{Binom}}{\overline{\overline{\text{coeff}}}} \Big($ ______ $\Big)$. And $\left[\!\left[\begin{smallmatrix} 70 \\ 42 \end{smallmatrix} \right]\!\right] = \left[\!\left[\begin{smallmatrix} T \\ N \end{smallmatrix} \right]\!\right]$, where $T =$ ______ $\neq 70$, and $N =$ ______.

For the two essay questions, carefully TYPE*, double-or-triple–spaced, grammatical solns. I suggest* LaTeX*, but other systems are ok too.*

A2: **i** On a 7×7 chessboard, 22 rooks are placed. Prove there exists a ***friendly*** 4-set of rooks. [I.e, on 4 distinct rows and on 4 distinct columns. Shorthand: You may use ***clump*** for "friendly 4-set".] Illustrate the concepts in your proof with *large, useful* Pictures. [*Hint:* PHP]

ii On a 7×7 chessboard, 23 rooks are placed. Prove: ***<u>Either</u>*** *there exists a friendly 5-set, or a disjoint-pair of friendly 4-sets.* [An n-set of rooks is ***friendly*** if the rooks lie on n distinct rows, and n distinct columns. Shorthand: You may use ***double-clump*** for "*disjoint-pair of friendly 4-sets*".]

STRONGER: Prove there *always* exists a double-clump, unconditionally.

A3: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

$$\dagger: \quad b_{n+2} := 7b_{n+1} - 10b_n, \quad \text{for } n = 0, 1, \ldots.$$

Use induction to prove, for each natnum k, that

$$\ddagger: \quad b_k = 5^k - 2^k.$$

Further: Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $b_{n+2} := \mathbf{S} b_{n+1} - \mathbf{P} b_n$, for arbitrary real-number coefficients **S** and **P**?

End of Home-A

A1:	___ ___ ___	140pts
A2:	___ ___	90pts
A3:	___ ___	60pts
Total:	___ ___ ___	290pts