

Plex
MAA4402 2838

Class-A

Prof. JLF King
Wedn 15Feb2023

NB. For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

Let **holom** abbreviate “holomorphic”, and **harm.fnc** abbreviate “harmonic function”.

A1: Short answer. Show no work.

a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is March 30, 2023.

Circle: Yes True **Résumé material!**

b The point $p := -3 + 2i$ goes, under stereographic projection, to (x, y, z) on the RS, where

$x =$ _____, $y =$ _____, $z =$ _____.

c Cross-ratio $[z, 2-i, \infty, 3i] = \frac{az+b}{cz+d}$ where

$a =$ _____, $b =$ _____, $c =$ _____, $d =$ _____.

d Write holomorphic $h(x + iy) = u(x, y) + iv(x, y)$.

Then: Sum $3u + 5v$ is harmonic. AT AF Nei

A prod. of two harm.fncs is harmonic. AT AF Nei

If functions $f, g: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are each harmonic, then sum $H(x + iy) := f(x, y) + ig(x, y)$ is holom. AT AF Nei

Fnc $\alpha(x + iy) := x^2 + [2xyi] - y^2$ is holomorphic. T F

e $\sum_{n=0}^{\infty} [i/3]^n =$ _____ + $[i \cdot$ _____].

f Subset $G \subset \mathbb{C}$ is the union of an open set with a closed set. If G is connected, then G is path-connected. T F

OYOP: In grammatical English *sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.

For a fnc h of form $h(z) = \frac{az+b}{cz+d}$, let $\text{Det}(h)$ mean

$\text{Det} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. E.g.,

$$g(z) = \frac{\overline{[3-i]} - 2z}{[4+zi] \cdot 2}$$

has $\text{Det}(g)$ meaning the determinant of $\begin{bmatrix} -2 & 3+i \\ 2i & 8 \end{bmatrix}$.

A2: Define LFT $V_P(z) := [z - P]/[1 - \overline{P}z]$, where complex number P has $|P| \neq 1$.

Thus $\text{Det}(V_P) =$ _____ . Its inverse-LFT is

$[V_P]^{-1}(z) = \frac{az+b}{cz+d}$, where number $b =$ _____.

Essay: The unit-circle is $S := \{u \in \mathbb{C} \mid |u| = 1\}$. Prove that V_P maps S into S . [Hint: Complex number ω is in S IFF product $\omega \overline{\omega}$ equals *What?*]

Map V_P sends the center of S to itself: T F

A1: _____ 155pts

A2: _____ 55pts

Total: _____ 210pts

NAME: _____

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor.”

Signature: _____