

Plex
MAA4402 2838

Class-A

Prof. JLF King
Wedn 15Feb2023**NB.** For short-answer: Write **DNE** if the object does not exist or the operation cannot be performed. NB: $\mathbf{DNE} \neq \{\} \neq 0$.Let **holom** abbreviate “holomorphic”, and **harm.fnc** abbreviate “harmonic function”.**A1:** Short answer. Show no work.

a

Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is March 30, 2023.

Circle: Yes True Résumé material!

b

The point $p := -3 + 2i$ goes, under stereographic projection, to (x, y, z) on the RS, where

$$x = \dots, y = \dots, z = \dots$$

c

Cross-ratio $[z, 2-i, \infty, 3i] = \frac{az+b}{cz+d}$ where

$$a = \dots, b = \dots, c = \dots, d = \dots$$

d

Write holomorphic $h(x+iy) = u(x, y) + iv(x, y)$.Then: Sum $3iu + 5v$ is harmonic: **AT** **AF** **Nei**A prod. of two harm.fncs is harmonic. **AT** **AF** **Nei**If functions $f, g: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are each harmonic, then sum $H(x+iy) := f(x, y) + ig(x, y)$ is holom. **AT** **AF** **Nei**Fnc $\alpha(x+iy) := x^2 + [2xyi] - y^2$ is holomorphic. **T** **F**

e

$$\sum_{n=0}^{\infty} [i/3]^n = \dots + [i \cdot \dots].$$

f

Subset $G \subset \mathbb{C}$ is the union of an open set with a closed set. If G is connected, then G is path-connected. **T** **F**OYOP: In grammatical English **sentences**, write your essay on every 2nd line (usually), so I can easily write between the lines.For a fnc h of form $h(z) = \frac{az+b}{cz+d}$, let $\text{Det}(h)$ mean $\text{Det}([a \ b \ c \ d])$. E.g,

$$g(z) = \frac{[3-i] - 2z}{[4+zi] \cdot 2}$$

has $\text{Det}(g)$ meaning the determinant of $\begin{bmatrix} -2 & 3+i \\ 2i & 8 \end{bmatrix}$.**A2:** Define LFT $V_P(z) := [z - P]/[1 - \bar{P}z]$, where complex number P has $|P| \neq 1$.Thus $\text{Det}(V_P) = \dots$. Its inverse-LFT is $[V_P]^{-1}(z) = \frac{az+b}{cz+d}$, where number $b = \dots$.**Essay:** The unit-circle is $S := \{u \in \mathbb{C} \mid |u| = 1\}$. Prove that V_P maps S into S . [Hint: Complex number ω is in S IFF product $\omega\bar{\omega}$ equals *What?*]Map V_P sends the center of S to itself: **T** **F****A1:** _____ 155pts**A2:** _____ 55pts**Total:** _____ 210pts

NAME: _____

HONOR CODE: *I have neither requested nor received help on this exam other than from my professor.*

Signature: _____