Proof-based course MAS ?*!? **Prereq-ℵ** Prof. JLF King *Touch*: 11Jan2026

Welcome: This is a practice prereq exam.
The purpose of this Prereq is to help you self-evaluate if you have the prerequisite knowledge for a proof-based course.

ℵ1: On your own sheets of paper, please write (double-spaced) proofs of the following, in complete English sentences. *Do **not** restate the problem.*

Bin1 Define binary operation "♡", on pair of reals, by $x \heartsuit y := xy + y$. Prove or disprove [i.e if false, give an explicit counterexample]: "♡" is associative.

On the set of postive real numbers, define "★", a binary operation, by $a \bigstar b := a^{\log(b)}$. Prove or disprove: "★" is associative.

Ind1 Define a sequence $\vec{\boldsymbol{b}} = (b_0, b_1, b_2, \dots)$ by $b_0 := 0$ and $b_1 := 3$ and

†: $$b_{n+2} := 9b_{n+1} - 18b_n\,, \quad \text{for } n = 0, 1, \dots.$$

Use induction to prove, for each natnum k, that

‡: $$b_k = 6^k - 3^k\,.$$

Further: Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 6 and 3 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n$, for arbitrary real-number coefficients **S** and **P**?

Ind2 Let $L(n) := 2^n$. And let $R(n) := n^2$. By induction on n, prove that $\forall n \in \mathbb{N}: \quad L(n) > R(n)$. *Explicitly* prove the base case. *Explicitly* state the induction implication, then prove that it holds for each $n \in \mathbb{N}$.

D Using set-builder notation, define

$\text{PRIMES} := \{n \in \text{WHAT} \mid \text{Conditions on } n\}$,

using some of the symbols

such that, if, then, and, or, not, 0 1 2 . . .
$\forall\ \exists\ \nexists\ \in\ \mathbb{N}\ +\ =$

and **avoiding** "*factor(s), divides, is-a-multiple, splits, irreducible, composite, Gcd, Lcm . . .*".

Rel1 On $\mathbb{Z}_+$, write $x \,\$\, y$ IFF $xy < 0$. So \$ is *Circle*
Transitive: *T F*. **Symm.**: *T F*. **Reflex.**: *T F*.

On $\mathbb{Z}$, say that $x \nabla y$ IFF $x - y \le 1$. Then ∇ is:
Trans.: *T F*. **Symm.**: *T F*. **Reflex.**: *T F*.
(Be *careful* on both parts!)

ℵ2: Math-Greek alphabet: Please write the **two** missing data of lowercase/uppercase/name. Eg:

"iota: ____ α: ____ B: ____." You fill in: ι I A *alpha* β *beta*.

Ω: ____ Ψ: ____ H: ____

σ: ____ γ: ____ λ: ____

theta ____ rho ____ delta ____ mu ____

ℵ3: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

ℓ1 The **slope** of line $3[y-5] = 2[x-2]$ is ____.
Point $(-4, y)$ lies on this line, where $y=$ ____.

ℓ2 Line $y = [M \cdot x] + B$ owns points $(4, 3)$ and $(-2, 5)$. Hence $M=$ ____ and $B=$ ____.

ℓ3 Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2, 1)$. So $M=$ ____ and $B=$ ____.

q1 The solutions to $3x^2 = 2 - 2x$
are $x =$ ____.

q2 The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$
are $y =$
____.
[*Hint:* Apply the Quadratic Formula to y^2.]

e1 $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}} =$ ____. $\log_{64}(16) =$ ____.

(continued. . .)

i1 Let $y = f(x) := \left[2 + \sqrt[5]{x}\right]/3$. Its inverse-function is $f^{-1}(y)=$ ____.

i2 Let $g(x) := x^3 - x$. Then $g^{-1}(6)=$ ____
and $[g^{-1}]'(6)=$ ____.

sg2 For natural number K, the sum

$\sum_{n=3}^{3+K} 4^n$ equals ____________________.

F Quadratic $6x^2 + 5x - 4 = [Ax - \alpha] \cdot [Bx - \beta]$, for numbers $A=$ ______, $\alpha=$ ______; $B=$ ______, $\beta=$ ______.

End of Prereq-ℵ

ℵ1:	___ ___ ___	110pts
ℵ2:	___ ___	20pts
ℵ3:	___ ___ ___	165pts
Total:	___ ___ ___	295pts

NAME: ____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____