

Sets and Logic
MHF3202 17HE

Class-A

Prof. JLF King
Friday, 09Feb2024**A4:** Short answer. Show no work. Write LARGE.Write DNE if the object does not exist or the operation cannot be performed. NB: $\text{DNE} \neq \{\} \neq 0$.**a** Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is **Mon., 25 Mar., 2024**.

Circle: Yes True Résumé material!

b $B^E = \sum_{j=0}^{59} \binom{59}{j} \cdot 4^{2j}$, for posints $B = \underline{\dots}$ & $E = \underline{\dots}$.**c** Write the free vars in each of these expressions.

$$\exists n \in \mathbb{N}: f(n) \in \bigcup_{\ell=p-4}^{p+7} \underbrace{\{x \in \mathbb{Z} \mid \ell \cdot n \equiv_5 x^2\}}_{E2} \underbrace{\dots}_{E1}.$$

E3: $\underline{\dots}$. E2: $\underline{\dots}$. E1: $\underline{\dots}$.

d LBolt: $\text{GCD}(70, 42) = \underline{\dots} \cdot 70 + \underline{\dots} \cdot 42$.So (LBolt again) $G := \text{GCD}(70, 42, 60) = \underline{\dots}$ and
 $\underline{\dots} \cdot 70 + \underline{\dots} \cdot 42 + \underline{\dots} \cdot 60 = G$.**e** The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] $\underline{\dots}$ many possibilities.This number *also* equals the number-of-ways of picking K candies from T many types of candy, where $K = \underline{\dots} \notin \{1, 6\}$ and $T = \underline{\dots} \notin \{1, 4\}$.OYOP: In grammatical English *sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.**A5:** An *Lmino* (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N , let \mathbf{R}_N denote the $3 \times N$ board: I.e.,  is the \mathbf{R}_5 board. Prove:Theorem: When N is odd, then board \mathbf{R}_N is not Lmino-tilable.You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.When N is even, our \mathbf{R}_N has exactly

many Lmino-tilings.

A4: _____ 105pts**A5:** _____ 60pts**Total:** _____ 165pts

NAME: _____

Energetic Proof-is-my-Middle-Name Student

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: _____