Staple!

Ord: ___ ___

Sets and Logic MHF3202 1079 | **Class-A** | Prof. JLF King Friday, 29Sep2023

A4: Short answer. Show no work. Write LARGE.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$600 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Mon., 25 Mar., 2024.

Circle: **Yes** **True** ***Résumé material!***

___ b Compute the real $\alpha =$ ⌞. . . . . ⌟ such that

$$*: \quad 3^{\alpha} \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \quad = \quad \sum_{j=0}^{2023} \binom{2023}{j} 8^j \, .$$

[*Hint:* The Binomial Theorem]

___ c The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ⌞. . . . . ⌟ many possibilities.

This number *also* equals the number-of-ways of picking K candies from T many types of candy, where $K =$ ⌞. . . . . ⌟ $\notin \{1, 6\}$ and $T =$ ⌞. . . . . ⌟ $\notin \{1, 4\}$.

___ d Rewrite $x \in \left[F \cup [\bigcap_{k \in B} G_k] \right]$ without $\cup, \cap$, by using only ∃ st. ∀ : ∈ ∋ ∨ ∧] [x F G k B,

as ⌞. . . . . ⌟.

___ e In $\left[5x^2 + 4y + z^3 + 7\right]^{20}$, compute these coeffs:

$\text{Coeff}(x^6 z^8) =$ ⌞. . . . . ⌟.

$\text{Coeff}(y^5 z^6) =$ ⌞. . . . . ⌟

[An integer, or a product of powers and multinomial-coeffs.]

___ f On a K-elt set Ω, the number $\#_K$ of **reflexive symmetric** binrels is ⌞. . . . . ⌟.

In particular, $\#_5 =$ ⌞. . . . . ⌟.

OYOP: *In grammatical English* ***sentences****, write your essay on every 2nd line* (usually)*, so I can easily write between the lines.*

A5: For $K = 0, 1, 2, \ldots$, define sum

$$\mathcal{S}_K := \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{K \cdot [K+1]} \overset{\text{note}}{=\!=} \sum_{n=1}^{K} \frac{1}{n \cdot [n+1]} \, .$$

Find a closed-form [no summation sign, nor dot-dot-dot] for $\mathcal{S}_K$. Prove your formula correct by induction on K.

A4:	___ ___ ___	120pts
A5:	___ ___	55pts
Total:	___ ___ ___	175pts

NAME: ⌞. . . . . ⌟
Energetic *Proof-is-my-Middle-Name* Student

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ⌞. . . . . ⌟