Staple!

Ord: ___ ___

Sets and Logic MHF3202 17HE | **Class-A** | Prof. JLF King Monday, 03Oct2022

OYOP: *In grammatical English* ***sentences****, write your essay on every 2nd line* (usually)*, so I can easily write between the lines.*

A4: Short answer. Show no work. Write LARGE.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ a Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is mid-March, 2023.
Circle: **Yes** **True** ***Résumé material!***

___ b Compute the real $\alpha =$ ⌞.....⌟ such that

$$*: \quad 3^{\alpha} \cdot \sum_{k=0}^{4050} \binom{4050}{k} 2^k \quad = \quad \sum_{j=0}^{2022} \binom{2022}{j} 8^j \,.$$

[*Hint:* The Binomial Theorem]

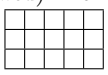
___ c Note that GCD$(55, 33, 15) = 1$. Find particular integers S, T, U so that $55S + 33T + 15U = 1$:
$S =$ ⌞.....⌟, $T =$ ⌞.....⌟, $U =$ ⌞.....⌟.
[*Hint:* GCD$\big($GCD$(55, 33), 15\big) = 1$.]

___ d On a K-elt set Ω, the number $\#_K$ of **reflexive symmetric** binrels is ⌞.....⌟.
In particular, $\#_5 =$ ⌞.....⌟.

On a 3-set, there are ⌞.....⌟ many equiv.relations.

___ e The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ⌞.....⌟ many possibilities.

This number *also* equals the number-of-ways of picking K candies from T many types of candy, where $K =$ ⌞.....⌟ $\notin \{1, 6\}$ and $T =$ ⌞.....⌟ $\notin \{1, 4\}$.

A5: An ***Lmino*** (pron. "ell-mino") comprises three squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3 \times N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{R}_N$ is <u>not</u> Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *<u>labeled</u>* pictures.

When N is <u>even</u>, our $\mathbf{R}_N$ has exactly ⌞.....⌟ *many Lmino-tilings.*

A4:	___ ___ ___	110pts
A5:	___ ___	55pts
Total:	___ ___ ___	165pts

NAME: ⌞.....⌟

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor or TA."*

Signature: ⌞.....⌟