

## Does Zero = One ?

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### Erroneousification

Here are various “proofs”<sup>♥1</sup> that zero equals one. Please find the errors/gaps in each.

**“Proof” e1.** Note that  $\frac{64}{16}$  equals 4, justified by cancelling the sixes,

$$\frac{64}{16} = \frac{\cancel{6}4}{\cancel{1}6} = \frac{4}{1} \stackrel{\text{note}}{=} 4.$$

Nothing is special about 4, and so

$$3 = \frac{39}{13} = \frac{\cancel{3}9}{\cancel{1}3} = \frac{9}{1} \stackrel{\text{note}}{=} 9.$$

Dividing gives  $1 = 3$ . Subtract 1 from each side, then halve, to obtain  $0 = 1$ . ♦

**No Man Clay Ture.** Henceforth, I call these vignettes **Poofs**; when you look at the argument more closely, it just disappears.

**Poof e2.** Write  $0 = 1$ . So  $0 = 1$ . Done. ♦

**Poof e3.** Write  $0 = 1$ . Multiplying by  $-6$  gives  $0 = -6$ . Adding 3 gives  $3 = -3$ , so squaring gives  $[3]^2 = [-3]^2$ ; that is,  $9 = 9$ . Which is true! **QED** ♦

**Poof e4.** The  $9 \times 9$  checkerboard can **not** be tiled by dominos, since 9 is odd. **Neither** can the  $7 \times 7$  checkerboard (same reason), hence  $7 = 9$ . Subtracting gives  $0 = 2$ . Now divide by 2. ♦

**Poof by induction, e5.** For  $k = 10^n$ , we prove that  $k = 10 \cdot k$ .

**Base case.** Well,  $0 = 10 \cdot 0$ , so done.

**Induction.** Assume  $k = 10 \cdot k$  for some  $k = 10^n$ . Multiplying by 10 gives  $10k = 100k$ . Plug in  $10^n$ ; so  $10 \cdot 10^n = 100 \cdot 10^n$ . Thus  $10^{n+1} = 10 \cdot 10^{n+1}$ . So the proposition holds for  $k = 10^{n+1}$ . **QED** ♦

**Corollary to e5.** Plug in  $k = 10^2$ ; so  $100 = 10 \cdot 100$ . Now subtract 100 to get  $0 = 900$ . Divide by the weight of a 900 pound Gorilla. So  $0 = 1$ . (And the Gorilla is on quite a diet.) ♦

**Poof A.** Square-rooting equality  $\frac{-1}{1} = \frac{-1}{-1}$  gives

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{-1}{-1}} \stackrel{\text{which}}{\implies} \frac{\sqrt{-1}}{1} = \frac{1}{\sqrt{-1}}.$$

Cross multiply to solve:

$$[\sqrt{-1}]^2 = 1^2 \implies -1 = 1 \implies 0 = 1. \quad \blacklozenge$$

**Poof B.** Factoring,  $a^2 - a^2 = [a + a][a - a]$ . Hence

$$a[a - a] = 2a[a - a] \implies a = 2a \implies 1 = 2 \implies 0 = 1. \quad \blacklozenge$$

**Poof C.** Consider  $\int \frac{1}{t} dt$ . Integrate by parts, using

$$u := \frac{1}{t} \text{ and } v := t. \text{ Thus } du = \frac{-1}{t^2} dt \text{ and } dv = dt.$$

Applying  $\int u dv = uv - \int v du$  yields

$$\int \frac{1}{t} dt = \frac{1}{t}t - \int \frac{-1}{t^2} \cdot t dt = 1 + \int \frac{1}{t} dt.$$

Subtracting  $\int \frac{1}{t} dt$  from each side gives  $0 = 1$ . ♦

**Poof D.** Letting  $N := 4x$ , we can write

$$4x \cdot x = \overbrace{x + x + x + \dots + x}^{N \text{ times}}.$$

Differentiate w.r.t  $x$ . Thus  $\frac{d}{dx}(4x^2) \stackrel{\text{note}}{=} 8x$  equals

$$\frac{d}{dx}(\overbrace{x + x + \dots + x}^{N \text{ times}}) = \overbrace{1 + 1 + 1 + \dots + 1}^{N \text{ times}} \stackrel{\text{note}}{=} N.$$

Thus  $8x = N \stackrel{\text{def}}{=} 4x$ . Since this eqn  $8x = 4x$  holds for all  $x$ , we get  $8 = 4$ . Halving twice gives  $1 = 0$ . ♦

<sup>♥1</sup>Arguments (e1) and (D) are my modifications of “puns” I’ve seen. Arguments (e2)–(e5), I’ve graded more times than I can count.

I do not know the authors of the other fantasies...

**“Pf” E.** Define  $s := \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \dots$

This righthand sum converges, by the **Alternating-series Test**, to a value  $s$  satisfying  $1 > s > 1 - \frac{1}{2}$ .

In particular,  $s \neq 0$ . Gather this sum *four-terms-at-a-time*, so as to write  $s$  as

$$\begin{aligned} & \left[ \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \dots \right] \\ & + \left[ -\frac{1}{2} - \frac{1}{6} - \frac{1}{10} - \dots \right] \\ & + \left[ -\frac{1}{4} - \frac{1}{8} - \frac{1}{12} - \dots \right] \end{aligned}$$

The top two long-brackets sum to

$$\sum_{D=1,3,5,7,\dots} \left[ \frac{1}{D} - \frac{1}{2D} \right] \stackrel{\text{note}}{=} \sum_{D=1,3,5,7,\dots} \frac{1}{2D}.$$

The bottom long-bracket is  $\sum_{n=1}^{\infty} \frac{-1}{4n}$ . Doubling  $s$  gives

$$\begin{aligned} 2s &= \left[ \sum_{D=1,3,5,\dots} \frac{1}{D} \right] + \left[ \sum_{n=1}^{\infty} \frac{-1}{2n} \right] \\ &= \left[ \sum_{\substack{D \in \mathbb{Z}_+, \\ \text{with } D \text{ odd}}} \frac{1}{D} \right] + \left[ \sum_{\substack{E \in \mathbb{Z}_+, \\ \text{with } E \text{ even}}} \frac{-1}{E} \right] \\ &= \sum_{k=1}^{\infty} \frac{[-1]^k}{k}. \end{aligned}$$

Hence  $2s = s$ . From above,  $s \neq 0$ ; so dividing gives  $2 = 1$ . Thus  $1 = 0$ , as desired.  $\blacklozenge$

**Poof F.** Recall  $e^{i\pi} + 1 = 0$ ; so  $e^{i\pi} = -1$ . Hence  $e^{i\pi} = i^2$ . Taking logs (naturally, naturally),

$$i\pi = \log(i^2) = 2 \cdot \log(i).$$

Consequently,  $2\pi = \frac{4 \cdot \log(i)}{i}$ . So

$$2\pi = \frac{\log(i^4)}{i} = \frac{\log(1)}{i} = \frac{0}{i} = 0.$$

Dividing by  $\pi$  gives  $2 = 0$ . Hence  $1 = 0$ .  $\blacklozenge$

**Poof G.** The function  $e^z$  omits the number 0 from its range, so the composition  $e^{e^z}$  omits  $1 = e^0$  as well as 0. But compositions of entire functions are entire, and the **Little Picard-Theorem** says that a non-constant entire function omits at most one complex value from its range. This forces that  $0 = 1$ .  $\blacklozenge$