# Does Zero = One ?

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## Erroneousification

Here are various "proofs"[♡1] that zero equals one. Please find the errors/gaps in each.

*"Proof" e1.* Note that $\frac{64}{16}$ equals 4, justified by cancelling the sixes,

$$\frac{64}{16} = \frac{\not{6}4}{1\not{6}} = \frac{4}{1} \stackrel{\text{note}}{=\!=} 4\,.$$

Nothing is special about 4, and so

$$3 = \frac{39}{13} = \frac{\not{3}9}{1\not{3}} = \frac{9}{1} \stackrel{\text{note}}{=\!=} 9\,.$$

Dividing gives $1 = 3$. Subtract 1 from each side, then halve, to obtain $0 = 1$. ♦

**No Man Clay Ture.** *Henceforth, I call these vignettes* ***Poofs****; when you look at the argument more closely, it just disappears.*

*Poof e2.* Write $0 = 1$. So $0 = 1$. Done. ♦

*Poof e3.* Write $0 = 1$. Multiplying by -6 gives $0 = -6$. Adding 3 gives $3 = -3$, so squaring gives $[3]^2 = [-3]^2$; that is, $9 = 9$. Which is *true!* **QED** ♦

*Poof e4.* The 9×9 checkerboard can **not** be tiled by dominos, since 9 is odd. **Neither** can the 7×7 checkerboard (same reason), hence $7 = 9$. Subtracting gives $0 = 2$. Now divide by 2. ♦

[♡1] Arguments (e1) and (D) are my modifications of "puns" I've seen. Arguments (e2)–(e5), I've graded more times than I can count.

I do not know the authors of the other fantasies....

*Poof by induction, e5.* For $k = 10^n$, we prove that $\boxed{k = 10 \cdot k}$.

**Base case.** Well, $0 = 10 \cdot 0$, so done.

**Induction.** Assume $k = 10 \cdot k$ for some $k = 10^n$. Multiplying by 10 gives $10k = 100k$. Plug in $10^n$; so $10 \cdot 10^n = 100 \cdot 10^n$. Thus $10^{n+1} = 10 \cdot 10^{n+1}$. So the proposition holds for $k = 10^{n+1}$. **QED** ♦

*Corollary to e5.* Plug in $k = 10^2$; so $100 = 10 \cdot 100$. Now subtract 100 to get $0 = 900$. Divide by the weight of a 900 pound Gorilla. So $0 = 1$. (And the Gorilla is on quite a diet.) ♦

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*Poof A.* Square-rooting equality $\frac{-1}{1} = \frac{1}{-1}$ gives

$$\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}} \stackrel{\text{which}}{\Longrightarrow} \frac{\sqrt{-1}}{1} = \frac{1}{\sqrt{-1}}\,.$$

Cross multiply to solve:

$$\left[\sqrt{-1}\right]^2 = 1^2 \implies -1 = 1 \implies 0 = 1\,. \quad ♦$$

*Poof B.* Factoring, $a^2 - a^2 = [a + a][a - a]$. Hence

$$a[a-a] = 2a[a-a] \implies a = 2a \implies 1 = 2 \implies 0 = 1\,. \quad ♦$$

*Poof C.* Consider $\int \frac{1}{t}\,dt$. Integrate by parts, using

$$u := \frac{1}{t} \text{ and } v := t\,. \quad \text{Thus } du = \frac{-1}{t^2}\,dt \text{ and } dv = dt\,.$$

Applying $\int u\,dv = uv - \int v\,du$ yields

$$\int \frac{1}{t}\,dt = \frac{1}{t}t - \int \frac{-1}{t^2} \cdot t\,dt = 1 + \int \frac{1}{t}\,dt\,.$$

Subtracting $\int \frac{1}{t}\,dt$ from each side gives $0 = 1$. ♦

*Poof D.* Letting $N := 4x$, we can write

$$4x \cdot x = \overbrace{x + x + x + \ldots + x}^{N \text{ times}}\,.$$

Differentiate w.r.t $x$. Thus $\frac{d}{dx}(4x^2) \stackrel{\text{note}}{=\!=} 8x$ equals

$$\frac{d}{dx}\Big(\overbrace{x + x + \ldots + x}^{N \text{ times}}\Big) = \overbrace{1 + 1 + 1 + \ldots + 1}^{N \text{ times}} \stackrel{\text{note}}{=\!=} N\,.$$

Thus $8x = N \stackrel{\text{def}}{=\!=} 4x$. Since this eqn $8x = 4x$ holds for all $x$, we get $8 = 4$. Halving twice gives $1 = 0$. ♦

*"Pf" E.* Define $s := \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \ldots$. This righthand sum converges, by the Alternating-series Test, to a value $s$ satisfying $1 > s > 1 - \frac{1}{2}$. In particular, $\boxed{s \neq 0}$. Gather this sum *four-terms-at-a-time*, so as to write $s$ as

$$\begin{aligned} &\left[\tfrac{1}{1} + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{7} + \tfrac{1}{9} + \tfrac{1}{11} + \ldots\right] \\ &+ \left[\tfrac{-1}{2} - \tfrac{1}{6} - \tfrac{1}{10} - \ldots\right] \\ &+ \left[- \tfrac{1}{4} - \tfrac{1}{8} - \tfrac{1}{12} - \ldots\right] \end{aligned}$$

The top two long-brackets sum to

$$\sum_{D=1,3,5,7,\ldots} \left[\frac{1}{D} - \frac{1}{2D}\right] \overset{\text{note}}{=\!=} \sum_{D=1,3,5,7,\ldots} \frac{1}{2D}.$$

The bottom long-bracket is $\sum_{n=1}^{\infty} \frac{-1}{4n}$. Doubling $s$ gives

$$\begin{aligned} 2s &= \Big[\sum_{D=1,3,5,\ldots} \frac{1}{D}\Big] + \Big[\sum_{n=1}^{\infty} \frac{-1}{2n}\Big] \\ &= \Big[\sum_{\substack{D \in \mathbb{Z}_+, \\ \text{with } D \text{ odd}}} \frac{1}{D}\Big] + \Big[\sum_{\substack{E \in \mathbb{Z}_+, \\ \text{with } E \text{ even}}} \frac{-1}{E}\Big] \\ &= \sum_{k=1}^{\infty} \frac{[-1]^k}{k}. \end{aligned}$$

Hence $2s = s$. From above, $s \neq 0$; so dividing gives $2 = 1$. Thus $1 = 0$, as desired. ♦

*Poof F.* Recall $e^{i\pi} + 1 = 0$; so $e^{i\pi} = -1$. Hence $e^{i\pi} = i^2$. Taking logs (naturally, naturally),

$$i\pi = \log(i^2) = 2 \cdot \log(i).$$

Consequently, $2\pi = \frac{4 \cdot \log(i)}{i}$. So

$$2\pi = \frac{\log(i^4)}{i} = \frac{\log(1)}{i} = \frac{0}{i} = 0.$$

Dividing by $\pi$ gives $2 = 0$. Hence $1 = 0$. ♦

*Poof G.* The function $e^z$ omits the number 0 from its range, so the composition $e^{e^z}$ omits $1 = e^0$ as well as 0. But compositions of entire functions are entire, and the Little Picard-Theorem says that a non-constant entire function omits at most one complex value from its range. This forces that $0 = 1$. ♦

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