Staple!

Team: ____

Abstract Algebra MAS4301 4864 — **Z-Home** — Prof. JLF King, *Touch*: 6May2016

Hello. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to each question.

General instructions are on the CHECKLIST.

Z1: Please write a proof for #24Page132. Isomorphic *as rings*? (*Jog:* $\sqrt{2}$-gp and matrix-gp iso.)

Z2: Please write a proof for #26Page132. (*Jog:* The quaternion 4-gp is not iso. to $\mathbb{D}_4$.)

Z3: Prove #34Page90. Pictures will help. (*Jog:* A gp with exactly one nt-proper subgp looks like what?)

Z4: Please write a proof for #16Page148. [*Hint:* Lagrange's Thm.] (*Jog:* $a^{\varphi(N)} \equiv_N 1$.)

Z5: On the 4×4 TTT (TicTacToe) board, let G denote the TTT-automorphism group. This is the set of self-bijections of $[1..4] \times [1..4]$ which preserve all TTTs. A particular TTT-auto is the ***swizzle***, S: It exchanges each corner square with the central square that it (diagonally) touches; and it does The Right Thing (tm) on the edge squares.

__ __ a) Find a TTT-auto T which is **not** in the subgp $\langle \text{Isometries}, S\rangle$, yet $\langle \text{Isometries}, S, T\rangle$ is all of G .

__ __ b) Compute the order of G. Find a set of *involutions* which generates G. What is a *minimum cardinality* generating set for G?

Compute the center of G; what is its order?

__ __ c) For $K = 1, 2, \ldots$, let G_K denote the TTT-automorphism gp of the $K \times K$ board. Writing K as $K = 2H$ or $K = 2H + 1$, as K is even or odd, compute the order of G_K in terms of H. How many *really different* (i.e, non-isomorphic) first moves are there?

What is the smallest generating set that you can find for G_K?

__ __ d) On the 4×4×4 TicTacToe board (Qubic), what is a 3-dim'al analog of The Swizzle? How many *really different* first moves are there? (ExtraCredit: Give a generating set for the gp TTT-autos, G_{Qubic}.)

Z6: Turnstile: It has 20 ***tokens*** in an oval-track, as well as a ***spinner*** that reverses the order of 4-consecutive tokens at the 12o'clock position.

i) Show that every pattern is obtainable. [*Hint:* Obtain an adjacent transposition.]

ii) Let $N := \#\text{Tokens}$. For odd $N>4$, show that a transposition is NOT obtainable. Are all even patterns obtainable?

iii) The ***width*** of the spinner is $W=4$. What can you say for general $W \geqslant 3$ and various (all?) N?

End of Z-Home

Z1:	__ __	35pts
Z2:	__ __	35pts
Z3:	__ __	65pts
Z4:	__ __	25pts
Z5:	__ __ __	135pts
Z6:	__ __ __	135pts
Total:	__ __ __	430pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

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Ord: ....