

Note. The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to the essay question.

General instructions are on the CHECKLIST. If you find an unfamiliar term, look in “Notes & Hints” at the end of this P-S. Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than .9797... Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g, “ $1/a+b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

Z1: Show no work. Questions (a)–(b) refer to these three vectors/points in \mathbb{R}^3 :

$$\mathbf{w} := (2, 1, -2), \quad \mathbf{v} := (5, 1, 2), \quad \mathbf{a} := (3, -1, 3).$$

a Compute the norm, $\|\mathbf{w}\| =$ _____, and the cross-product, $\mathbf{v} \times \mathbf{w} =$ _____, and the projection of \mathbf{v} in the \mathbf{w} -direction, $\text{Proj}_{\mathbf{w}}(\mathbf{v}) =$ _____.

b The distance from the origin to Plane($\mathbf{w}, \mathbf{v}, \mathbf{a}$) equals _____.

c List the six vertices of some regular octahedron H :
So $\cos(\text{dihedral } \angle \text{ of } H) =$ _____.

d The quadratic surface $x^2 + y^2 + 6x + 4y + 2z = z^2 - 11$ is a _____.
[Hint: Complete the square for each of x, y, z .]

e In \mathbb{R}^4 , let \mathbb{L}_0 be the line passing through the origin and the point $Q = (1, 2, 3, 4)$. Let \mathbb{L}_1 be the line $t \mapsto (2, -1, 0, 1) + t(5, 0, 1, 2)$. The (orthogonal) distance between lines \mathbb{L}_0 and \mathbb{L}_1 equals _____.

f Let \mathbb{L}_3 be the line $3 - x = \frac{y-6}{2} = z - 3$. Let \mathbb{L}_2 be the line passing through the origin and the point $Q = (4, -8, -4)$. Then $\text{Dist}(\mathbb{L}_2, \mathbb{L}_3) =$ _____.

g Compute the **arclength** of the curve $\mathbf{w}(t) := t^2\hat{i} + [\cos(t) + t\sin(t)]\hat{j} + [\sin(t) - t\cos(t)]\hat{k}$ from $t = 0$ to $t = \pi$.
Arclength = _____.

h For a posreal B , define helix $\mathbf{r}_B(t) := \cos(t)\hat{i} + \sin(t)\hat{j} + B \cdot t \cdot \hat{k}$. Compute the radius-of-curvature $\text{RoC}_B =$ _____.

i Let \mathbf{A} and \mathbf{R} be the surf-acc and radius of some nice planet. Then the golden-snitch speed is: $S_{\text{gold}} =$ _____.
The *period* of the snitch is $L_{\text{gold}} =$ _____.
The snitch goes around the equator in the same direction that the Earth turns. How many times a day do you have to duck? Same \approx _____.
If it goes in the opposite dir.? Opp \approx _____.

Z2: (Let \mathbf{A} denote the surf-acc of Earth, in in/sec².) The Boston Science Museum has a curvy-funnel (AoS vertical; with diam= 4ft). Pushing a button releases a ball-bearing (BaB) which rolls angled into the funnel; its path looks like a planetary orbit, because the funnel is shaped correctly. Let $g(r)$ denote the depth of the funnel at radius r (each in inches). Design $g()$ st., for each r , if BaB is rolling at the correct speed to stay at the radius= r circle, then the toward-AoS component of force is

$$\mathbf{A} \cdot \frac{U}{r^2},$$

where $U := 24 \cdot \text{in}^2$.
So $g(r) =$ _____ and
 $\text{BaB-speed}(r) =$ _____.
(Answers *must* be ITOF \mathbf{A} and U ; your units *must* be correct. **I may add a small part to this question**, to be announced in class.)

Notes & Hints. (Be aware that Marston Science Library has mathematical dictionaries.) There is a convenient way to place a cube in 3-space. Since the octahedron is the dual of a cube, there is an easy way to place an octahedron in space. The dihedral angle at an edge of a polyhedron, is the angle between the two *faces* which meet at that edge. (You’ll need to think about exactly what this means.)

End of Z-Home

Z1: ___ ___ ___ 180pts

Z2: ___ ___ 75pts

Total: ___ ___ ___ 255pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____