Staple!

Ord: __ __

DiffyQ MAP2302 | **Z: Individual Optional Project** | Prof. JLF King Friday, 04Dec2015

This IOP is due **2PM, Thurs., 10Dec2015**, slid *completely* under my office door, 402 Little Hall.

This sheet is "Page 1/N", and you've labeled the rest as "Page 2/N",..., "Page N/N".

Your 3 essay(s) must be TYPESET*, and Double or Triple spaced. Use the Print/Revise cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper. Do* **not** *restate the problem; just solve it.*

Z1: *How does an ice cube melt?*

Your essay should have many large, well-drawn pictures, of the various situations.

Terms to research/look-up. ***"Specific heat"*** of a substance; in the sense of "heat capacity per unit mass of a material". ***"Heat of fusion"*** and ***"latent heat"*** of a substance as it melts from solid to a liquid at the same temperature. ***"Thermal conductivity"*** of a substance. For those of these quantities that you need, give letter-names to them. It is possible that the values differ for ice and for water [so you may need two names]. *Use the simplification* that thermal conductivity is independent of temperature.

You may need the density of water, and the density of ice; if so, give them letter names. Or perhaps you only need a name for the *ratio* of their densities.

Carefully define any other terminology that you need; give citations to Wikipedia pages, or elsewhere, where appropriate. Aside: Both you and I may use tmp to abbreviate "temperature".

Ice. Use ***hunk*** for the ice "cube" [shapes will be discussed in a moment] under discussion.

A hunk at tmp 0° [all tmps are in centigrade] is suspended[♡1] in a bath of water at teMperature $\mathcal{M} > 0°$.

As the hunk melts, assume that it keeps its shape [a ball remains a ball; a cube remains a cube] as its volume, $V(t)$, decreases. Let $\mathcal{V}_0$ denote the initial volume.

[♡1] Since we don't want the complexity of the hunk floating and touching a surface of the container, you may imagine this entire experiment takes place on the Space Station, in 0-gravity. Or that some force repels the hunk from the container surface.

Water-bath of ∞ volume. Suppose that there is so much water that its tmp does not appreciably change as the ice melts. The water is well-mixed, so that as soon as some 0°-ice phase-transitions ["melts"] into 0°-water, that them there water is whisked away and does not affect the hunk's melting. [Not to worry; that water's tmp will be raised to $\mathcal{M}$ by the water-bath.]

a Using *letters* [which you have defined in a table] for the various ice&water constants and initial-values, write the DEs for $V(t)$ both in the case that the hunk is a ball, and is a cube. Number these DEs as (1_{Ball}) and (1_{Cube}).

b If you can solve these DEs [does SoV work?], then do so. Compute the ***meltdown time*** [the time for the hunk to completely melt into water] symbolically.

c Now substitute in the actual values for ice&water for, say, an ice ball-or-cube of the size used in a drink. Look up the actual melting times [or do the experiment yourself with real ice]; how does your result compare to actual times?

Water-bath of finite volume. Let $\mathcal{W}_0$ be the initial volume of water. Let $W(t)$ denote the volume of water at time t; so $W(t)$ increases with t, while $V(t)$ decreases. *However*, the sum

$$\text{Vol}(t) \quad := \quad V(t) + W(t)$$

is not constant, as ice is less-dense than water [which is why ice floats in water].

The ice is always at 0°. With $\mathcal{M}_0$ the initial water tmp, let $M(t)$ denote the water's tmp at time t.

d In this model, the water's tmp decreases, as the ice melts. Note also that, now, as the ice melts, the newly-extant water needs to absorb enough heat-energy to become the temperature of the ambient water [the tank is well-mixed].

Using letters, write down the system of DEs for the

$$V(t),\ W(t),\ M(t)$$

triple, in the case that the hunk is indeed a cube.

Z2: Suppose that $P(), Q(), G()$ are $\mathbf{C}^\infty$-fncs, and $y = y(t)$ is a soln to DE

$$*: \qquad y'' + P(t)y' + Q(t)y \;=\; G(t)\,.$$

Argue carefully that y *must* be a $\mathbf{C}^\infty$-fnc.

Z3: Consider U.F $y = y(t)$ satisfying IVP

$$ty'' \;+\; y' \;+\; ty \quad = \quad 0\,,$$

where $y(0) = 1$. Let $Z(s) := \big[\mathcal{L}(y)\big](s)$ denote the Laplace transform of y.

___ I Use our properties of $\mathcal{L}()$ to derive a *first-order* separable DE that $Z()$ satisfies.

___ II Use SoV to solve for $Z()$.

___ III The inverse transform $\big[\mathcal{L}^{-1}\big(Z\big)\big](t)$ may not be easy to compute, but tell me whatever you can about it.

End of Z: Individual / Optional / Project

Z1:	___ ___ ___	125pts
Z2:	___ ___	85pts
Z3:	___ ___ ___	105pts
Total:	___ ___ ___	315pts

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:

Folks, I have had a great time working with you this Semester. Stop by next semester to "Talk Math".

Cheers, Prof. Sieve-brain