Staple!

Sets and Logic
MHF3202 8768

Prac-prereq-

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Hello. *This practice prereq is to help you self-evaluate on material you should review.*

Without reviewing anything, no books, no internet, take 65 minutes to do as many of the problems below that you can. *Then relax for a while...* Then use books/internet to grade yourself.

Short-answer: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

ℓ1 The **slope** of line $3[y-5] = 2[x-2]$ is ______.
Point $(-4, y)$ lies on this line, where $y=$ ______.

ℓ2 Line $y = [M \cdot x] + B$ owns points $(4,3)$ and $(-2,5)$. Hence $M=$ ______ and $B=$ ______.

ℓ3 Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2,1)$. So $M=$ ______ and $B=$ ______.

q1 The solutions to $3x^2 = 2 - 2x$ are $x =$ ______.

q2 The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$ are $y =$ ______.
[*Hint:* Apply the Quadratic Formula to y^2.]

e1 $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}} =$ ______. $\log_{64}(16) =$ ______.

i Let $y = f(x) := \left[2 + \sqrt[5]{x}\right]/3$. Its inverse-function is $f^{-1}(y) =$ ______.

id1 Suppose g is a fnc with g' never zero. Let h be the inverse-fnc of g. In terms of h, g, g' and x, write a formula for $h'(x) =$ ______.
[*Hint:* The Chain rule. NOTE: h is **NOT** $1/g$.]

id2 Let $g(x) := x^3 + x$. Then $g^{-1}(10) =$ ______
and $[g^{-1}]'(10) =$ ______.

de2 On those x where $\sin(x) > 0$, define $B(x) := [\sin(x)]^x$. Its derivative is
$B'(x) =$ ______.
[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

dc1 Below, f and g are differentiable fncs with

$$f(2) = 3, \quad f(3) = 5, \quad f'(2) = 19, \quad f'(3) = 17,$$
$$g(2) = 11, \quad g(3) = 13, \quad g'(2) = \tfrac{1}{2}, \quad g'(3) = 7,$$
$$f(5) = 43, \quad g(5) = 23, \quad f'(5) = 41, \quad g'(5) = 29\,.$$

Define the composition $C := g \circ f$. Then
$C(2) =$ ______ ; $C'(2) =$ ______.

Please write each answer as a product of numbers; **do not** multiply out. [*Hint:* The Chain rule.]

sg1 Compute the sum of this geometric series: $\sum_{n=3}^{\infty} [-1]^n \cdot [3/5]^n =$ ______.

sg2 For natural number K, the sum $\sum_{n=3}^{3+K} 4^n$ equals ______.

sg3 $\sum_{n=1}^{\infty} r^n = \frac{5}{8}$. So $r=$ ______ or **DNE**.
[*Hint:* The sum starts with n at **one**, not zero.]

sc The series $\sum_{k=1}^{\infty} \frac{[-1]^k}{\log(k)}$ (circle one): Diverges, Converges absolutely, Converges conditionally.

pf Partial-fraction decomposition:

$$\frac{x+1}{x^2+x-2} = \_\_\_\_\_\_ + \_\_\_\_\_\_ .$$

Short-answer: Sets&Logic stuff:

B Binomial coefficient $\binom{7}{4} =$ ______ $=$ ______.

C Relation $\mathbf{R}$ is a binrel on set $\mathbb{Z}_+$, defined by $x\mathbf{R}y$ IFF $x^2 = 5y$.
Assertion "*Relation* $\mathbf{R}$ *is reflexive*" is *T F*
Assertion "*Relation* $\mathbf{R}$ *is antireflexive*" is *T F*

As On $\mathbb{Z}$, the statment "operator $+$ is *associative*" means: *For all* ______ *in* ______ *it is the case that*
______.