Staple!

Sets and Logic
MHF3202 8768

Prac-prereq-V

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Hello. *This practice exam is longer than what the actual prereq-knowledge exam will be.*

V2: Show no work. *NOTE*: The **inverse-fnc** of g, often written as g^{-1}, is *different* from the **reciprocal fnc** $1/g$. E.g, suppose g is invertible with $g(-2) = 3$ and $g(3) = 8$: Then $g^{-1}(3) = -2$, yet $[1/g](3) \stackrel{\text{def}}{=} 1/g(3) = 1/8$.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

ℓ1 The **slope** of line $3[y-5] = 2[x-2]$ is ________.

Point $(-4, y)$ lies on this line, where y= ________.

ℓ2 Line $y = [M \cdot x] + B$ owns points $(4,3)$ and $(-2,5)$. Hence M= ________ and B= ________.

ℓ3 Line $y = Mx + B$ is orthogonal to $y = \frac{1}{3}x + 2$ and owns $(2,1)$. So M= ________ and B= ________.

q1 The solutions to $3x^2 = 2 - 2x$ are $x =$ ________.

q2 The four solutions to $[y-2] \cdot y \cdot [y+2] = -1/y$ are $y =$ ________.

[*Hint:* Apply the Quadratic Formula to y^2.]

e1 $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}}$= ________. $\log_{64}(16)$= ________.

i Let $y = f(x) := \left[2 + \sqrt[5]{x}\right]/3$. Its inverse-function is $f^{-1}(y)$= ________.

id1 Suppose g is a fnc with g' never zero. Let h be the inverse-fnc of g. In terms of h, g, g' and x, write a formula for $h'(x)$= ________.

[*Hint:* The Chain rule. NOTE: h is **NOT** $1/g$.]

id2 Let $g(x) := x^3 + x$. Then $g^{-1}(10)$= ________

and $[g^{-1}]'(10)$= ________.

dq $\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\sin(3z)}{\cos(z+1)}\right) = \frac{f(z)}{g(z)}$ where

$f(z)$= ________

and $g(z)$= ________.

de1 For $x > 0$, let $B(x) := x^x$. Its derivative is

$B'(x)$= ________.

[*Hint:* How is y^z, for $y > 0$, defined in terms of the exponential fnc?]

a For $x > 0$, let $B(x) := x^{\sin(x)}$. Hence its derivative is $B'(x) = B(x) \cdot M(x)$, where $M(x)$ equals

________.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

de2 On those x where $\sin(x) > 0$, define $B(x) := [\sin(x)]^x$. Its derivative is

$B'(x)$= ________.

[*Hint:* How is y^z, for $y > 0$, defined ITOf the exponential fnc?]

dc1 Below, f and g are differentiable fncs with

$$\begin{array}{llll} f(2) = 3, & f(3) = 5, & f'(2) = 19, & f'(3) = 17, \\ g(2) = 11, & g(3) = 13, & g'(2) = \frac{1}{2}, & g'(3) = 7, \\ f(5) = 43, & g(5) = 23, & f'(5) = 41, & g'(5) = 29\,. \end{array}$$

Define the composition $C := g \circ f$. Then
$C(2) =$ ________ ; $C'(2) =$ ________.

Please write each answer as a product of numbers; **do not** multiply out. [*Hint:* The Chain rule.]

sg1 Compute the sum of this geometric series:
$\sum_{n=3}^{\infty} [-1]^n \cdot [3/5]^n =$ ________.

sg2 For natural number K, the sum $\sum_{n=3}^{3+K} 4^n$ equals

________.

sg3 $\sum_{n=1}^{\infty} r^n = \frac{5}{8}$. So r= ________ or **DNE**.

[*Hint:* The sum starts with n at **one**, not zero.]

di1 With $f(t) := \int_{t^5}^{7t} \cos(\sin(x))\,\mathrm{d}x$, then $f'(t)$ equals

________.

Simplified, $f'(0) =$ ________.

[*Hint:* Chain rule and Fund. Thm of Calculus.]

sc The series $\sum_{k=1}^{\infty} \frac{[-1]^k}{\log(k)}$ (circle one): Diverges, Converges absolutely, Converges conditionally.

pf Partial-fraction decomposition:

$$\frac{x+1}{x^2+x-2} = \underline{\qquad\qquad} + \underline{\qquad\qquad}.$$