Staple!

Ord: ____ ____

Plex
MAA4402 8436

IOP-Z

Prof. JLF King
Tuesday, 02Nov2021

Plexologists: This IOP (Individual Optional Project) (Individual Project C) is due by 2:30PM, Thursday, 09Dec2021. slid completely under my office door, 402 Little Hall (in the northeast corner).

Important: This Problem-Sheet is the *first page* of your double-stapled write-up and the essays must be typeset. If possible, use alignment in displays. Diagrams may be hand-drawn.

Z1: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

a Let $h(z) := \exp(\frac{1}{z}) \cdot \exp(\frac{1}{3z})$. Then $\operatorname{Res}_{z=0}\big(h(z)\big) =$ ⌞.....⌟. And $\operatorname{Res}_{z=0}\big(z \cdot h(z)\big) =$ ⌞.....⌟.

b Consider entire function $G(z) := \sum_{n=0}^{\infty} a_n \cdot [z-8]^n$. Then

$$\operatorname{Res}_{z=0}\Big(\frac{G(z)}{z^4}\Big) \;=\; \sum_{n=K}^{\infty} a_n \cdot W_n\,, \quad \textit{where}$$

$K =$ ⌞.....⌟ $\in \mathbb{N}$ and $W_n =$ ⌞.....⌟.
[Each W_n is a number. You may use binomial coefficients in expressing your W_n.]

c For $N \in [2\,..\,\infty)$, number $V_N =$ ⌞.....⌟ where

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + [N+1]x + N^2}\,\mathrm{d}x \;=\; \frac{2\pi}{V_N}\,.$$

Essay questions:

Z2: A function f is ***nice*** if f is holomorphic on open ball $\boldsymbol{B} := \mathrm{Bal}_1(0)$, *and* $f(0) = 3$. A radius $r<1$ is ***good*** if *some* point w on circle $\mathsf{C}_r := \mathrm{Sph}_r(0)$ satisfies $\big|f(w)\big| = 3$.

i Produce a *specific* nice fnc g with *bad* radius $b<1$.

ii *The actual problem*: Prove that each nice fnc h has some positive radius $R<1$ for which: *Every* radius $r<R$ is good.
[*Hint:* Use ideas related to the Maximum-modulus principle. You may want the Intermediate-value theorem applied to some real-valued function.]

Z3: Use a keyhole contour to carefully compute

$$J := \int_0^{\infty} \frac{x^{1/4}}{[x+1]^3}\,\mathrm{d}x\,,$$

showing all the steps. Prove that the asymptotic contribution of the integral along the two circles, is zero. Use the branch of Arg (as in class) that takes values in $[0, 2\pi)$.
[*Hint:* As models, use the two examples in PlexNotes, as well as the GCIF to compute the appropriate residues.]

Please PRINT your name and ordinal. Ta:

Ord:
⌞.....⌟

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:
⌞.....⌟