Staple!

Team: ____

Abstract Algebra MAS4301 4864 — **Y-Home** — Prof. JLF King *Touch*: 6May2016

Hello. Essays violate the CHECKLIST at *Grade Peril*!

Y1: Show no work.

___ ___ a #**14** P 191. $\mathrm{Ord}(14 + \langle 8\rangle) =$ ______.

___ ___ b #**26** P 191. $G/H \cong$ ______.

___ ___ c #**30** P 191. $U(165) \cong$ ______, ______, ______, ______.

___ ___ d There exist groups P, A, B with $P{\times}A \cong P{\times}B$, yet $A \not\cong B$. Circle one: **True** **False**

___ ___ e+ Let $G := (\mathbb{Z}_2, +)$ and $H := \mathrm{Aut}(G{\times}G{\times}G)$. Then $\mathrm{Ord}(H) =$ ______. This H is abelian: **True** **False**

___ ___ f+ Group $\mathbb{S}_{11}$ has elts (in std.CN)

$$\alpha := (1\ 2\ 3)\,(4\ 5\ 6\ 7)\,(8\ 9\ 10\ 11)\ ;$$
$$\beta := (1\ 3\ 2)\,(4\ 5\ 7\ 6)\,(8\ 9\ 11\ 10)\ .$$

Both α and β are in $\mathbb{A}_{11}$:	**True**	**False**
Elts α and β are conjugate in $\mathbb{S}_{11}$:	**True**	**False**
Elts α and β are conjugate in $\mathbb{A}_{11}$:	**True**	**False**

___ ___ g #**10** P 255. In $\mathbb{Z}_7$: ______, ______, ______, ______.

Y2: #**12** P 240; a ring *with* unit. (*Jog:* Non-abel, 16 elts)

Y3: Prove #**56** P 191, with particular attention to (d). (*Jog:* Properties of commutator subgp.)

Y4: Generalize example #**16** P 208 to:

THEOREM: For primes $p < q$ such that *FOO*, the only order-pq group is cyclic. Argue analogously to class: Setup | There exists an order-7 elt in G | There is exactly one order-7 subgp, K | $\mathfrak{N}_G(K) = G$ | Centralizer $\mathfrak{C}_G(K)$ is all of G. Make *FOO* ASiAP.

i List four pairs $p<q$ where your thm applies (call such, a ***good*** pair), four bad pairs, and an ∞-family where even the conclusion fails.

ii Use Dirichlet's Thm (stated in #**35** P 227) to show there are ∞-ly many primes q so that $5<q$ is good. Generalize.

iii Use Bertrand's Postulate to prove that there are arbitrarily long chains $p_0 < p_1 < \cdots < p_L$, where each pair $p_{j-1} < p_j$ is good.

Y5: α Let G denote TTT-$\mathrm{Aut}(4{\times}4)$; use **C** for a corner/center cell, and **E** for an edge cell. After FM=**C** (First Move at **C**), how many *Really Different* second moves are there? –and how many of each *RD*-type? Ditto for FM=**E**. Compute

$$\mathrm{Stab}(\mathbf{C}) \quad\text{and}\quad \mathrm{Stab}(\mathbf{E})\,.$$

Explicitly use Orb-Stab Thm to check.

β With G acting on itself by conjugation, either compute the size of each conjugacy-class, or of each pt-centralizer. Use the *class-eqn* (P403) to compute $\mathrm{Ord}(\mathrm{Z}(G))$.

γ With $G :=$ TTT-$\mathrm{Aut}(4{\times}4{\times}4)$, i.e Qubic, do parts ($\alpha$) and ($\beta$), until you run out of stamina. Can you generalize?

End of Y-Home

Y1:	___ ___ ___	135pts
Y2:	___ ___	15pts
Y3:	___ ___	65pts
Y4:	___ ___ ___	125pts
Y5:	___ ___ ___	155pts
Total:	___ ___ ___	495pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ______

Ord: ______

Ord: ______