Staple! Ord: __ __

Sets and Logic
MHF3202 8768

IndividualOP-Y

Prof. JLF King
Thur, 13Apr2017

This IOP is due **2PM, Thur., 20Apr2017**, slid *completely* under my office door, 402 LITTLE HALL. This sheet is "Page $1/N$", and you've labeled the rest as "Page $2/N$"... "Page N/N".

Y1: *Show no work.*

__ __ **a** Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.

Define $\omega\,\mathbf{U}\,\lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].
Define $\omega\,\mathbf{I}\,\lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.
So "$\mathbf{U}$ *is an equiv-relation*" is: T F
So "$\mathbf{I}$ *is an equiv-relation*" is: T F

__ __ **b** Let $\mathcal{P}_\infty$ denote the family of all ***co-finite*** subsets of $\mathbb{N}$. That is, a subset $S \subset \mathbb{N}$ is an *element* of $\mathcal{P}_\infty$ IFF $\mathbb{N} \smallsetminus S$ is finite. Define relation $\bowtie$ on $\mathcal{P}_\infty$ by: $A \bowtie B$ IFF $A \cap B$ is infinite.

Stmt "*This* $\bowtie$ *is an equivalence-relation*" is: T F

Your 2 essay(s) must be TYPESET, *and Double or Triple spaced. Use the* Print/Revise *cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper. Do* **not** *restate the problem; just solve it.*

Y2: [Here, **"graph"** means *"non-void finite simple graph"*.]

A graph M is ***gluing-good*** if, for all graphs H, S having M as a subgraph, necessarily

$$\dagger: \qquad \mathcal{P}_G(x) = \frac{\mathcal{P}_H(x) \cdot \mathcal{P}_S(x)}{\mathcal{P}_M(x)},$$

whenever G is an M-gluing of H with S.

i For each posint N, prove that the complete graph K_N *is* gluing-good.

ii For $N = 3, 4, \ldots$, show that the path-graph P_N is *not* gluing-good, as follows. Exhibit graphs H_N and S_N, that can be glued [give a specific gluing] over P_N, to produce a graph G_N such that $\mathcal{P}_{G_N}(x) \neq \dfrac{\mathcal{P}_{H_N}(x) \cdot \mathcal{P}_{S_N}(x)}{\mathcal{P}_{P_N}(x)}$.

iii Generalize: Prove that if M is *not* a complete graph, then M *not* gluing-good.

Y3: [For free: Union Thm: *A countable union of countable-sets is countable.* Also, Finite-subset Thm: *The collection of* ***finite*** *subsets of a countable set, is countable.* If needed, use $\mathcal{P}_{\mathrm{Fin}}(S)$ for the collection of *finite* subsets of a set S, and use $\mathcal{P}_\infty(S)$ for the collection of **infinite** subsets of S.] Below, a ***blip*** is an *infinite* set of natnums. A ***family***, $\mathcal{F}$, is a set [not a multiset] of blips, i.e, $\mathcal{F} \subset \mathcal{P}_\infty(\mathbb{N})$.

a Suppose, $\forall B,C \in \mathcal{F}$, that $[B \neq C] \implies [B \cap C = \varnothing]$. Construct, with proof, an *injection* $g{:}\mathcal{F} \hookrightarrow \mathbb{N}$, to conclude that every such family, $\mathcal{F}$, must only be countable.

b Weaken the hypothesis on $\mathcal{F}$ to: *For all* $B,C \in \mathcal{F}$: $[B \neq C] \Rightarrow |B \cap C| \leqslant 1$. Prove $\mathcal{F}$ is tiny; only countable.

Weaken further to $[B \neq C] \Rightarrow |B \cap C| \leqslant 2$, yet still prove $\mathcal{F}$ countable. Now weaken to $[B \neq C] \Rightarrow |B \cap C| \leqslant 3$, and prove $\mathcal{F}$ is only countable. *Generalize!*

c [Challenging/Creative; A converse.] Construct a *specific* uncountable family $\mathcal{U}$, so that: *For all distinct* $B,C \in \mathcal{U}$*: Intersection* $B \cap C$ *is* finite.

End of IndividualOP-Y

Y1:	__ __	30pts
Y2:	__ __ __	000pts
Y3:	__ __ __	000pts
Total:	__ __	30pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord:

Folks, I've had a wonderful time Problem-Solving with you. Do consider my Combinatorics course in Fall 2017. Stop by in future semesters for Math/chess/coffee.

Cheers, Coun-SeLo-r King