Differential Eqns
MAP2302

Y-Class

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Y1: Show no work.

z If $\lim_{x\to 0+} 8/x$ equals ∞, then $\lim_{x\to 0+} 5/x$ is Circle:

Prof. King's beret *Brine* $\rotatebox{180}{5}$

a U.F. $x = x(t)$ satisfies $\boxed{2x^{(3)} + 5x^{(2)} - x \;=\; 0}$.

Then $\mathsf{Y} \coloneqq \begin{bmatrix} x \\ x' \\ x'' \end{bmatrix}$ satisfies $\mathsf{Y}' = \mathsf{M}\cdot\mathsf{Y}$, where M is this 3×3 matrix of numbers:

M =

.

b Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$x' + 4x - y = 0,$$
$$y' + 2x + 7y = 0.$$

It can be written as $\mathsf{Z}' = \mathsf{R}\cdot\mathsf{Z}$,
where $\mathsf{Z} \coloneqq \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ and R is matrix ______________.

Characteristic poly of R is $\wp_{\mathsf{R}}(z)=$ ______________

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for *numbers* $\alpha=$ __________ and $\beta=$ __________.

c Fnc $y_{\alpha,\beta}(t) = \alpha e^{At} + \beta e^{Bt} + P\cdot\sin(t) + Q\cdot\cos(t)$ is the general soln to

$$*:\qquad 3y'' + 4y' + y = \cos(t),$$

with numbers $A=$ ____, $B=$ ____, $P=$ ____, $Q=$ ____.

d Matrices $\mathsf{B}, \mathsf{U}, \mathsf{N}$ are 3×3, with B invertible and N nilpotent. [Use $\mathbf{I}$ for the 3×3 identity matrix.]

Matrix BNB^{-1} *is nilpotent*:	*AT*	*AF*	*Nei*
Each entry of $e^{t\mathsf{N}}$ *is a polynomial*:	*AT*	*AF*	*Nei*
Matrix e^{N} *is nilpotent*:	*AT*	*AF*	*Nei*
N^2 *is the zero-matrix*:	*AT*	*AF*	*Nei*
Matrix $e^{[\mathsf{U}+\mathbf{I}]\mathsf{U}}$ *equals* $e^{\mathsf{U}} \cdot e^{\mathsf{U}^2}$:	*AT*	*AF*	*Nei*
Matrix $e^{[\mathsf{U}^2]}$ *equals* $[e^{\mathsf{U}}]^2$:	*AT*	*AF*	*Nei*

Y2: Show no work.

e Consider linear DiffOp

$$\mathsf{V}(y) \;:=\; ty'' - [1+t]y' + y\,.$$

Verify [for yourself] that $\mathsf{V}(Y_0) = 0$ and $\mathsf{V}(Y_1) = 0$, where $Y_0 := \mathsf{e}^t$ and $Y_1 := 1+t$. Their Wronskian is $\mathcal{W}(Y_0, Y_1) =$ .... Then VoP tells us that

$y_{\alpha,\beta} :=$ ....

is the general soln to $\mathsf{V}(y_{\alpha,\beta}) = 3t^2$.

f With $f(x) := \mathsf{e}^{7x}$ and $g(x) := \mathsf{e}^{4x}$, then $[f \circledast g](5) =$ ....

With $\mathbf{1}()$ the constant-1 fnc and $F(x) := \sin(5x)$, then, convolution $[\mathbf{1} \circledast F](x) =$ ....

End of Y-Class

Y1:	___ ___ ___	140pts
Y2:	___ ___	95pts
Total:	___ ___ ___	235pts