Staple!

Ord: ___ ___

Sets and Logic
MHF3202 7860

Class-Y

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I give permission for Prof. King to email my grades to my `ufl.edu` *eddress.* Circle: *Yes* *No*

Y1: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ **a** The basic theory of infinite cardinals was developed by [Circle]: **Abel Alladi Avogadro Bernstein Bertrand Cantor Cauchy Dedekind Fraenkel Gauss Hilbert Russell Shakespeare Zermelo**

He started this work in latter half of the [Circle]:

1400s 1500s 1600s 1700s 1800s 1900s

___ **b** The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{23}$ is ________.

[Write your answer as a product of powers and multinomial-coeffs.]

___ **c** For $Y := \{1, 2, 3, 4\}$, consider $f : Y \to \mathcal{P}(Y)$ by

$$f(1) := \{3, 4\}, \qquad f(2) := Y,$$
$$f(3) := \varnothing, \qquad f(4) := \{1, 4\}.$$

The set $B := \{x \in Y \mid f(x) \not\ni x\}$ is $\{$ ________ $\}$.

___ **d** An explicit bijection $\psi : \mathbb{Z} \hookrightarrow\!\!\!\!\twoheadrightarrow \mathbb{N}$ is this:

If $n \geqslant 0$, then $\psi(n) :=$ ________.

If $n < 0$, then $\psi(n) :=$ ________.

___ **e** The number of ways of having 4 objects from 8 types is $\left[\!\left[\begin{matrix} 4 \\ 8 \end{matrix} \right]\!\right] \frac{\text{Binom}}{\text{coeff}} \Big($ ____ $\Big) \frac{\text{Integer}}{\text{numeral}}$ ________.

And

$\left[\!\left[\begin{smallmatrix} 4 \\ 8 \end{smallmatrix} \right]\!\right] = \left[\!\left[\begin{smallmatrix} K \\ L \end{smallmatrix} \right]\!\right]$, where $K =$ ________ $\neq 4$, and $L =$ ________.

___ **f** Let $\mathcal{P}_\infty$ denote the family of all ***co-finite*** subsets of $\mathbb{N}$. That is, a subset $S \subset \mathbb{N}$ is an *element* of $\mathcal{P}_\infty$ IFF $\mathbb{N} \smallsetminus S$ is finite. Define relation $\bowtie$ on $\mathcal{P}_\infty$ by: $A \bowtie B$ IFF $A \cap B$ is infinite.

Stmt "*This $\bowtie$ is an equivalence-relation*" is: *T* *F*

___ **g** Relation $\mathbf{R}$ is a binrel on set $\mathbb{N}$, defined by $x\mathbf{R}y$ IFF $x^2 = 5y$.

Assertion "*Relation $\mathbf{R}$ is reflexive*" is *T* *F*

Assertion "*Relation $\mathbf{R}$ is antireflexive*" is *T* *F*

OYOP: *In grammatical English **sentences**, write your essay on every **third** line* (usually), *so that I can easily write between the lines. Do* **not** *restate the question.*

Y2: Between sets $\mathbf{A} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $f : \mathbf{A} \hookrightarrow \mathbf{\Omega}$ and $g : \mathbf{\Omega} \hookrightarrow \mathbf{A}$, defined by

$$f(z) := 3z \quad \text{and} \quad g(\beta) := \beta + 5.$$

The S-B thm produces a set $Y \subset g(\mathbf{\Omega}) \subset \mathbf{A}$ so that, letting $X := \mathbf{A} \smallsetminus Y$, function $\theta : \mathbf{A} \hookrightarrow\!\!\!\!\twoheadrightarrow \mathbf{\Omega}$ is a *bijection*, where

$$*: \qquad \theta\rfloor_X := f\rfloor_X \quad \text{and} \quad \theta\rfloor_Y := g^{-1}\rfloor_Y.$$

i Prove, for these particular injections, that there is only *one* set Y which makes $(*)$ a bijection. [Does an orbit-picture help?]

ii Compute $\theta(56) =$ ________ and $\theta(83) =$ ________, drawing the appropriate part of (f, g)-orbit pictures.

End of Class-Y

Y1:	___ ___ ___	155pts
Y2:	___ ___	40pts
Total:	___ ___ ___	195pts