Staple!

Sets and Logic
MHF3202 2787

Optional-X

Prof. JLF King
31Aug2015

X1:	___ ___	95pts
X2:	___ ___ ___	145pts
Total:	___ ___ ___	240pts

Your 2 essay(s) must be TYPESET*, and Double or Triple spaced. Use the Print/Revise cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper. Do* **not** *restate the problem; just solve it.*

Due: By noon, on Monday, 09Dec2013, slid *completely* under my office door. Then please *email me* that you have handed-in a project.

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:

X1: Let $\mathbf{J} := [0,1]$. You may use, without proof, the Schröder-Bernstein thm and the following.

$\mathbf{a_1}$: $\mathbb{R} \asymp \{0,1\}^{\mathbb{N}}$. **$\mathbf{a_2}$:** $\mathbb{N}\times\mathbb{R} \asymp \mathbb{R}$.

$\mathbf{a_3}$: *For each three sets* Ω,B,D: $\Omega^{B\times D} \asymp \left[\Omega^B\right]^D$.

$\mathbf{a_4}$: *The set* $S := \mathbb{Q}\cap\mathbf{J}$ *is countable.*

Prove that $\mathbf{C}(\mathbf{J})$*, the set of continuous functions* $\mathbf{J}\to\mathbb{R}$*, is bijective with* $\mathbb{R}$. Cite each ($\mathbf{a}_i$) where you use it. Specify what Ω, B, D are, when you apply ($\mathbf{a_3}$). [*Note:* Does your proof split into easily-understood lemmas?]

X2: [For free: Union Thm: *A countable union of countable-sets is countable.* Also, Finite-subset Thm: *The collection of* ***finite*** *subsets of a countable set, is countable.* If needed, use $\mathcal{P}_{\text{Fin}}(S)$ for the collection of *finite* subsets of a set S, and use $\mathcal{P}_{\infty}(S)$ for the collection of **infinite** subsets of S.] Below, a ***blip*** is an *infinite* set of positive integers. A ***family*** $\{B_i\}_{i\in\mathbf{J}}$ is a set of *distinct* blips, i.e, $\forall i,k\in\mathbf{J}$: $[i\neq k] \Rightarrow [B_i\neq B_k]$.

a Suppose $\forall i,k\in\mathbf{J}$ that $[i\neq k]\Rightarrow[B_i\cap B_k=\varnothing]$. Construct, with proof, an *injection* $g:\mathbf{J}\hookrightarrow\mathbb{N}$, to conclude that this index-set $\mathbf{J}$ is only countable.

b Instead suppose $|B_i\cap B_k|\leqslant 1$, for each distinct index-pair $i,k\in\mathbf{J}$. Prove that $\mathbf{J}$ is only countable. Once solved, weaken the hypothesis to $|B_i\cap B_k|\leqslant 2$, yet still show $\mathbf{J}$ countable. Finally, weaken to $|B_i\cap B_k|\leqslant 3$, and prove that $\mathbf{J}$ is only countable. Can you generalize?

c [Challenging/Creative; Making $\mathbf{J}$ equal $\mathbb{R}$.] Construct a *specific* family $\{B_x\}_{x\in\mathbb{R}}$, that is, define a *specific* injection $h:\mathbb{R}\hookrightarrow\mathcal{P}_{\infty}(\mathbb{Z}_+)$, so that: *For each distinct index-pair $x,y\in\mathbb{R}$, the intersection $[B_x\cap B_y]$ is finite.*

Folks, I have had a great time working with you this Semester. Stop by next semester to "Talk Math".

Cheers, Prof. Sieve-brain