Staple!

Team X____

Sets and Logic
MHF3202 7860

Genuine-**Home-X**

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Touch: 21Feb2017

Due **BoC, Monday, 16Mar2015**, with *all team-members present*. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

This sheet is "Page $1/N$", and you've labeled the rest as "Page $2/N$",..., "Page N/N". Fill-in [large handwriting] *on this problem-sheet* all of your blanks.

X1: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

__ __ **a** Define fncs $G,P{:}[1\,..\,12]\circlearrowleft$, where $G(n)$ is the number of letters in the n^{th} Gregorian month [so $G(2) = 8$; the 2^{nd} month "February" has 8 letters], and $P(n) := 13 - n$.

The *set* of posints k with $G^{\circ k}(1) = G^{\circ k}(2)$ is ______________.

Let $f := P \circ G$. Then $f^{\circ 2}(11) =$ __________.

Statement "$P \circ G = G \circ P$" is (circle) *T* *F*

__ __ **b** $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. *T* *F*

$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. *T* *F*

For all sets Ω, *there exists a fnc* $f{:}\mathbb{R}\to\Omega$. *T* *F*

__ __ **c** [With $\mathcal{P}()$ the *powerset* operator, let $S :=$ 3-stooges.] Then $|\mathcal{P}(S)| =$ ______ and $|\mathcal{P}(\mathcal{P}(S))| =$ ______________.

OYOP: *Your 2 essay(s) must be* TYPESET, *and Double or Triple spaced. Use the* Print/Revise$\circlearrowleft$ *cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper. Do* **not** *restate the problem; just solve it.*

On the essays: *I ask that you do more than what I ask you to do.*

X2: For all natnums $k < n$, prove that $H_k \perp H_n$, where

$$H_k \quad := \quad 1 + 6^{[2^k]}\,.$$

[*Hint*: For each natnum m, define $G_m := -1 + 6^{[2^m]}$. Prove a divisibility relation among the Hs and the Gs, by induction. Then a common divisor of H_k and H_n must...]

Also, produce an index $\ell \in \mathbb{N}$ st. H_ℓ is not prime.

X3: In our Velleman text, solve problem **#12**P.277. Let $\mathbf{E}_n$ be the equilateral triangle with side-length 2^n. This $\mathbf{E}_n$ can be tiled in an obvious way by 4^n many little-triangles [copies of $\mathbf{E}_0$]; see picture P.277. The ***"punctured*** $\mathbf{E}_n$**"**, written $\widetilde{\mathbf{E}_n}$, has its topmost copy of of $\mathbf{E}_0$ removed.

A ***trapezoiD*** D comprises three copies of $\mathbf{E}_0$ glued together in a row, rightside-up, upside-down, rightside-up [see picture P.277].

PROVE: *For each* n, *board* $\widetilde{\mathbf{E}_n}$ *admits a trapezoid-tiling.* Call such, a ***"zoid-tiling*** of $\widetilde{\mathbf{E}_n}$**"**.

In case you want to generalize, let $\mathbf{B}_k$ be the equilateral triangle of sidelength k; so $\mathbf{E}_n$ is $\mathbf{B}_{2^n}$. Triangle $\mathbf{B}_k$ comprises k^2 little-triangles. Two nice extra credit things to think-about/prove: *For what values* k *does* $\mathbf{B}_k$ *admit a zoid-tiling? For which* k *does* $\widetilde{\mathbf{B}_k}$ *admit a zoid-tiling?*

Another possible direction: Generalize the Lmino tiling to three-dimensions, tiling the punctured $2^n \times 2^n \times 2^n$ board. What plays the role of the tile?

End of *Genuine*-Home-X

X1:	__ __	80pts
X2:	__ __ __	105pts
X3:	__ __	95pts
Ouch!, scratch work handed-in; OR Poorly stapled. :	__ __	−20pts
Total:	__ __ __	280pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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