Staple!

Team X____

Sets and Logic
MHF3202 8768

Home-X

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Touch: 4Aug2016

Due **BoC, Monday, 24Mar2014**, with *all team-members present*. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

X1: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

___ ___ a Define G:[1 .. 12]↻ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2nd month is "February". The only fixed-point of G is ______ . The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ______ .

___ ___ b We consider binrels on $\Omega :=$ Stooges $:= \{M, L, C\}$. There are ______ **Anti-reflexive** binrels, and ______ **Reflexive** binrels, and ______ **Symmetric** binrels. The number of **strict total-orders** is ______ .

___ ___ c Suppose **R** is a binrel on set Ω. Then statement "*Relation* $\mathbf{R} \circ \mathbf{R}^{-1}$ *equals* $\mathbf{R}^{-1} \circ \mathbf{R}$" is T F

OYOP: *Your 2 essay(s) must be* TYPESET, *and Double or Triple spaced. Use the* Print/Revise↻ *cycle to produce good, well thought out, essays. Start each essay on a* ***new*** *sheet of paper.*

Do ***not*** *restate the problem; just solve it.*

X2: Note $f(n) := \frac{1}{2}\cdot[15^n + 19^n]$ is an integer. Prove, for each *odd* $n \geqslant 3$, that $f(n)$ is composite. [*Hint:* Look at $f(n)$ mod something.]

[Are all the hypotheses necessary, or can some be weakened?]

X3: i Over a 29 day month, SeLoidian Bubba posts at least one soln per day, for a total of 45 solns. PROVE:

> *There is a period of consecutive days over which he posted exactly $g := 13$ solutions.*

[g for "**G**uaranteed".] NOTE: In your proof, let s_n denote the number of solns posted that month by the end of day n. By hyp., then,

$$1 \leqslant s_1 < s_2 < \ldots < s_{29} = 45\,.$$

Let $t_n := 13 + s_n$. *Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.* [*Hint:* You may find it easier to first show that g=12 is guaranteed. Then you'll see how to show that g=13 is guaranteed.]

ii *Generalize*: Replace 29 by D, replace 45 by P; we now consider posints with D < P . Give a formula for the *largest* value, call it Γ(D, P), for which your proof *guarantees* the values $g = 1, 2, 3, \ldots, \Gamma(\mathsf{D}, \mathsf{P})$.

iii For fixed D and P, let $\mathcal{M}(\mathsf{D}, \mathsf{P})$ be the *set* of guaranteed posints g. What can you tell me about the structure of $\mathcal{M}(\mathsf{D}, \mathsf{P})$? Conjectures? Proofs? Computer experiments? (*I* don't know the structure. What can you teach me?)

End of Home-X

X1:	___ ___	85pts
X2:	___ ___	75pts
X3:	___ ___ ___	105pts
Ouch!, scratch work handed-in; OR Poorly stapled. :	___ ___	−20pts
Total:	___ ___ ___	265pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ______

Ord: ______

Ord: ______