

X5: Short answer: Show no work. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. Use ‘Rv’ for “random variable”, and ‘Var’ for “variance”.

z The stmt “ $\forall N$ there exists a prime $p \in [N..2N]$ ” is called:

- Circle
- Prime Number Thm Riemann Hypothesis Chebyshev’s Thm
Eratosthenes’ Wild Guess Merten’s Thm Bertrand’s Postulate

a For $|x| < 1$, use the “ x trick” to compute

$$\sum_{n=0}^{\infty} nx^n = \dots \quad \& \quad \sum_{n=0}^{\infty} n^2 x^n = \dots$$

b Rv X is the number of flips till first head, where $P(\text{heads}) = \frac{1}{5}$. So $E(X) = \dots$. And $E(X^2) = \dots$.

Hence $\text{Var}(X) = \dots$.

c The discriminant of poly $h(z) := z^2 - z + 3$ is $B^2 - 4AC = \dots$. Let α and β be h ’s roots.

Set $\zeta := 4 + \alpha$. Its norm is $\mathcal{N}_h(\zeta) = \dots \in \mathbb{Z}$.

Its h -conjugate $\bar{\zeta} = \dots + \dots \alpha$. Finally,

$$[1/\zeta] = \dots + \dots \alpha. \text{ [All coeffs are rational.]}$$

d For posint N , let $\lambda(N)$ be the *smallest* $J \in \mathbb{Z}_+$ with $\forall x \in \Phi(N) : x^J \equiv_N 1$. Then

$$\lambda(3) = \dots, \lambda(5) = \dots \text{ and } \lambda(15) = \dots$$

Given posints K and L , write $\lambda(3^K \cdot 5^L)$ as product of prime-powers:

[Hint: The Primitive-root Thm is of use.]

Essay question: Write in complete sentences and also fill-in the blanks. Each essay starts a new page.

X6: Let $N := 561 \stackrel{\text{note}}{=} 3 \cdot 11 \cdot 17$. Note

*: For each prime $p \mid N : p-1 \mid N-1$.

Prove: $\boxed{\text{For each } k \perp N : k^{N-1} \equiv_N 1}$.

[Hint: Chinese Remainder Thm and Fermat’s Little Thm.]

Bonus: The 14th cyclotomic polynomial is $C_{14}(z) = \dots$

End of Class-X

X-Home:	_ _ _	365pts
X5:	_ _ _	120pts
X6:	_ _ _	65pts
Bonus:	_ _ _	10pts
Total:	_ _ _	550pts

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor (or his colleague).”
Name/Signature/Ord

Ord: _____