Sets and Logic MHF3202 8768 **Class-X** Prof. JLF King 29Mar2017

X1: Short answer. Show no work.

Write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

a For $G := \{1,2,3,4\}$, consider $f\colon G \to \mathcal{P}(G)$ by

$$f(1) := \{3,4\}, \qquad f(2) := G,$$
$$f(3) := \varnothing, \qquad f(4) := \{1,4\}.$$

The set $B := \{x \in G \mid f(x) \not\ni x\}$ is $\{$ ________________ $\}$.

b Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This $\approx$ is an equivalence-relation*" is: *T* *F*

c Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\mathbf{Y} := \mathbb{N}$, consider injections $f\colon \mathbf{X} \hookrightarrow \mathbf{Y}$ and $h\colon \mathbf{Y} \hookrightarrow \mathbf{X}$, defined by

$$f(x) := 3x \quad \text{and} \quad h(y) := y + 5.$$

Schröder-Bernstein produces a set $G \subset h(\mathbf{Y}) \subset \mathbf{X}$ st., letting $U := \mathbf{X} \smallsetminus G$, the fnc $\beta\colon \mathbf{X} \hookrightarrow\!\!\!\!\!\rightarrow \mathbf{Y}$ is a *bijection*, where

$$*: \qquad \beta|_U := f|_U \quad \text{and} \quad \beta|_G := h^{-1}|_G.$$

For <u>this</u> (f,h), the (U,G) pair is unique. Computing, $\beta(56)=$ ______ . $\beta(137)=$ ______ . $\beta^{-1}(603)=$ ______ .

d Define fncs $G,P\colon[1..12]\circlearrowleft$, where $G(n)$ is the number of letters in the n^{th} Gregorian month [so $G(2) = 8$; the 2^{nd} month "February" has 8 letters], and $P(n) := 13 - n$.

The *set* of posints k with $G^{\circ k}(1) = G^{\circ k}(2)$ is ________________________________ .

Let $f := P \circ G$. Then $f^{\circ 2}(11) =$ ____________ .

Statement "$P \circ G = G \circ P$" is [circle] *T* *F*

e The ***complete graph on N vertices***, K_N, has $\binom{N}{2}$ edges. The set of posints N for which K_N admits an Eulerian circuit is ________________________ .

f From the 300×200 game-board, cut-out (remove) the $(35,150)$-cell and one other cell at $P = (x,y)$. [Circle] those choices for P,

$(150,160)$, $(14,35)$, $(66,77)$, $(195,15)$, $(123,4)$

which, if removed, would leave a board that *definitely* can**<u>not</u>** be domino-tiled.

OYOP: *In grammatical English sentences, write your essay on every third line* (usually)*, so that I can easily write between the lines.*

X2: Let $\mathbf{J} := [3,7]$, an interval of reals. You may use, without proof, the Schröder-Bernstein thm and:

$\mathbf{a_1}$: $\mathbb{R} \asymp \{0,1\}^{\mathbb{N}}$. $\quad \mathbf{a_2}$: $\mathbb{N}\times\mathbb{R} \asymp \mathbb{R}$.

$\mathbf{a_3}$: *For each three sets Ω,B,D:* $\Omega^{B\times D} \asymp \left[\Omega^B\right]^D$.

$\mathbf{a_4}$: *The set $S := \mathbb{Q} \cap \mathbf{J}$ is denumerable.*

Prove that $\mathbf{C}(\mathbf{J})$, the set of continuous functions $\mathbf{J}\to\mathbb{R}$, is bijective with $\mathbb{R}$. Cite each $(\mathbf{a}_i)$ <u>where</u> you <u>use</u> it. Specify what Ω, B, D are, when you <u>apply</u> $(\mathbf{a_3})$. [*Note:* Split your proof into easily-stated lemmas.]

End of Class-X

X1: ___ ___ ___ 145pts

X2: ___ ___ 80pts

Total: ___ ___ ___ 225pts