

Plex MAA4402 8436 **Class-X** Prof. JLF King
 Wednesday, 17Nov2021

Read! the Notation! Use $C_r := Sph_r(0)$.

The expression $FUNCTION = u + iv$ means that u is the real-part of $FUNCTION$, and v is the imaginary-part.

$Res(\beta, q)$ is the residue of *function* $\beta()$ at the point q . Please simplify answers, when possible.

X1: Short answer. Show no work.

a Which action *loses pts*? circle *Writing-in-sentences*
Writing-t-different-from- + tiny - tiny - writing *Writing-LARGE*

b Fnc $\alpha(z) := \frac{\cosh(7z)}{[z-2]^2 z^2}$ has a $z=0$ pole of order .
 And $Res(\alpha, 0) =$.

c Let $J := \int_{C_2} z^4 \sin(\frac{7}{z}) dz$. So $J =$.

d Defining $\vec{a} = (a_0, a_1, \dots)$ by $\cos(2z) = \sum_{n=0}^{\infty} a_n z^n$, coefficient $a_4 =$. Let

$$\sum_{n=0}^{\infty} r_n z^n = \frac{1}{\cos(2z)}$$

be the reciprocal PS. Then $r_4 =$.
 [Hint: You can compute the reciprocal as we did in class.]

X2: Short answer. Show no work.

Let $f(z) := z^4 + 5z^2 + 4$. Reciprocal $H(z) := 1/f(z)$ has, in the upper half-plane, two poles p and q , where p lies closer to the origin than q .

So $Res(H, p) =$ and $Res(H, q) =$.

Our D -contour technique applies to H .

Thus $J := \int_{-\infty}^{+\infty} \frac{1}{x^4 + 5x^2 + 4} dx =$.

[Hint: Factor $f(z)$ as a product of two quadratics with integer coeffs.]

OYOP: In *grammatical English sentences*, write your essay on every 2nd line (usually), so I can easily write between the lines.

X3: α Precisely state the Gauss Mean-value thm, with all the hypotheses. State it as a *formal* theorem.

β Carefully prove the Gauss MVT, using the CIF.

X4: Consider an entire fnc $g = u + iv$ whose imaginary-part, v is bounded, e.g, $|v(z)| \leq 5$ for all z . Prove that g is constant. [Hint: Somehow prove that g is bounded, then apply SomebodyOrOther's thm. (Who's thm? State the theorem formally.) You may want to construct an auxiliary function from g .]

X1: 115pts

X2: 60pts

X3: 55pts

X4: 40pts

Total: 270pts

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: