Number Theory
MAS 4203

TakeHome-X

Prof. JLF King
Touch: 3Aug2016

Note. All variables range over the integers, unless otherwise specified. The exam is due *5PM on Sunday, 30Apr2000,* **slid under my office door,** room LIT402.

$X$1: Please FITBlank: Show no work (no partial credit).

a $\varphi(40) =$ ________. Let $\mathcal{C}$ be the set of mod-40 symm-residue classes which are *odd* and *coprime to* 10. Let $\mathcal{N} \subset \mathcal{C}$ comprise those residue classes D such that $\left(\frac{-10}{D}\right) = -1$. (That is negative ten, Folks.)

Then $\mathcal{N} = \{$ ________ $\}$.

b Let $f(x) := x^f2 - 2x + 11$ and let $M_1 := 7$, $M_2 := 49$ and $M_3 := 49 \cdot 2 = 98$. Compute a solution s_j to congruence

$$f(s_j) \equiv_{M_j} 0, \quad \text{where } \tfrac{-1}{2}M_j < s_j \leqslant \tfrac{1}{2}M_j.$$

$s_1 =$ ________; $s_2 =$ ________; $s_3 =$ ________.

c If $7^e \bullet\!\| [2000!]$, then $e =$ ________.

Note. For the following problem please carefully write up your solution on separate sheets of paper. Show all work –there *is* partial credit.

$X$2: Prove *Beatty's Theorem*, which is P.186#23.

1: Beatty's Theorem. *Suppose α and β are positive irrational reals such that*

$$\frac{1}{\alpha} + \frac{1}{\beta} = 1.$$

Then the following two sets of integers are disjoint,

$$A := \{\lfloor n\alpha \rfloor \mid n \in \mathbb{Z}_+\}, \qquad B := \{\lfloor n\beta \rfloor \mid n \in \mathbb{Z}_+\},$$

and their union is $\mathbb{Z}_+$. That is, (A, B) is a partition of the posints. ◊

[*Hint:* For each posint K, how big is the set $\{n \mid n\alpha < K\}$? Now do the same for β, and add the results; what integer is this sum? (Can you write this sum ITForm *Formula*(K)?) How does this sum change when you increment K to become $K+1$?]

$X$3: Please write a detailed solution to P.195#18. [*Note* You may use the Möbius inversion formula without proof, if you wish.]

$X$4: Create your own number theory problem, then solve it. Make this a "prove this" problem, rather than a "compute this" problem.

Bonus: Solve problem P.195#26 (having looked at #25). Note that the NZM "$\Phi_n(x)$" refers to a *polynomial*. It is related to, but is not the same as, our use of "Φ" in class.

$X$1:	___ ___ ___	110pts
$X$2:	___ ___ ___	120pts
$X$3:	___ ___ ___	110pts
$X$4:	___ ___ ___	110pts
Bonus:	___ ___	30pts
Total:	___ ___ ___	450pts

Please PRINT your name and ordinal. Ta:

________________________________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________

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