

X4: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a The author of our text is Circle: **DNE**
Archimedes Euler Fuchs Gauss Machen Mendez
Mueller Silverman Vaughn Velleman Whittle Zeitz

b Mimicking what we did in class: From the 300×200 game-board, cut-out (remove) the **(35, 150)**-cell and one other cell at $P = (x, y)$. Circle those choices for P ,
(150, 160), (14, 35), (66, 77), (195, 15), (123, 4)
 which, if removed, would leave a board that *definitely cannot* be domino-tiled.

c Both \sim and \bowtie are equiv-relations on a set Ω . Define binrels **I** and **U** on Ω as follows.
 Define $\omega \mathbf{U} \lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].
 Define $\omega \mathbf{I} \lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.
 So “**U** is an equiv-relation” is: **T** **F**
 So “**I** is an equiv-relation” is: **T** **F**

d Let \mathcal{P}_∞ denote the family of all *infinite* subsets of \mathbb{N} . Define relation \approx on \mathcal{P}_∞ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt “*This \approx is an equivalence-relation*” is: **T** **F**

e LBolt: $\text{Gcd}(70, 42) = \underline{\hspace{2cm}} \cdot 70 + \underline{\hspace{2cm}} \cdot 42$.
 So (LBolt again) $G := \text{Gcd}(70, 42, 60) = \underline{\hspace{2cm}}$ and
 $\underline{\hspace{2cm}} \cdot 70 + \underline{\hspace{2cm}} \cdot 42 + \underline{\hspace{2cm}} \cdot 60 = G$.

OYOP: *In grammatical English sentences, write your essay on every **third** line (usually), so that I can easily write between the lines. Do **not** restate the question.* Start each essay on a new sheet-of-paper. Please number the pages “1 of 57”, “2 of 57”... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

X5: On a 7×7 chessboard, 22 rooks are placed. Prove there exists a **friendly** 4-set of rooks. [I.e, on 4 distinct rows and on 4 distinct columns.] [*Hint: PHP*] Illustrate the concepts in your proof with *large, useful Pictures*.