Staple!

Ord: ___ ___

Calc 3 MAC2313 — **Prac-W** — Prof. JLF King, *Touch*: 4Aug2016

This is much longer than the actual exam.

W1: Short answer. Show no work.
Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ ___ **a** The plane through points $\mathbf{p} := (0, 1, -3)$, $\mathbf{q} := (5, 0, 1)$ and $\mathbf{r} := (2, 3, 1)$ can be written in form $Ax+By+Cz = T$, for numbers $A=$ ______, $B=$ ______, $C=$ ______, and $T=$ ______. The point on this plane which is closest to the origin is (______, ______, ______).

___ ___ **b** Write an eqn for the plane which is tangent to $x^2 - y^2 + z^2 = 0$ at the point $(3, 5, 4)$. Write the plane in the form $\alpha[x - x_0] + \beta[y - y_0] + \gamma[z - z_0] = 0$.
Plane: ______ $= 0$.

___ ___ **c** In $\mathbb{R}^3$, let S be the surface

$$6x - 3x^2 + 3 \quad = \quad 2y^2 + z^2 .$$

Vector $\mathbf{v}=$ ______ $\neq \mathbf{0}$ is $\perp$S at $Q := (0, 1, 1)$.
In form $A[x - x_0] + B[y - y_0] + C[z - z_0] = 0$, write an *equation* for the tangent plane to S at Q.
Eqn: ______
Have arranged that A, B, C are **integers** with no common factor; also, that $A \geqslant 0$.

___ ___ **d** The graph of $y = \frac{1}{12x} + x^3$, for $x \in (1, 2)$, has length= ______.

___ ___ **e** Compute the **arclength** of the curve $\mathbf{w}(t) := t^2\hat{\boldsymbol{\imath}} + [\cos(t) + t\sin(t)]\hat{\boldsymbol{\jmath}} + [\sin(t) - t\cos(t)]\hat{\mathbf{k}}$ from $t = 0$ to $t = \pi$.
Arclength= ______.

___ ___ **f** At the point $P := (1, 0)$, the *curvature* of curve $y = \ln(x)$ is ______.

___ ___ **g** Consider sets Ω and B and fnc $f{:}\Omega \to B$. Then
Graph(f)= ______ (Set-builder notation).

Suppose $g{:}\mathbb{R}^L \to \mathbb{R}^N$ is continuous. Then $\text{Dim}\big(\text{Graph}(g)\big) =$ ______.

For points $\mathbf{T}_k, \mathbf{S} \in \mathbb{R}^3$, the phrase "$\big[\lim_{k\to\infty} \mathbf{T}_k\big] = \mathbf{S}$" means $\lim_{k\to\infty}$ ______ $=$ ______.

W2: Short answer. Show no work.
Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

h Let $P_0 := (1, 3, 2\pi)$. Compute the gradient of $h(x, y, z) := x^2yz + y^3 \cdot \sin(z)$.
$[\nabla h](P_0) =$ ______.

i Consider a smooth $f{:}\mathbb{R}^2 \to \mathbb{R}$ and $P \in \mathbb{R}^2$. This P is a "***critical point*** of f" if [Imagine 2 blank lines].
The "***Hessian matrix*** of f, at P" is ______.

The ***second-derivative test*** for whether P is a *saddle-pt*, a *max-pt* or a *min-pt* of f, is: [Imagine 7 blank lines].

j On the circle $x^2 + y^2 = 1^2$, the max-point of $\Gamma(x, y) := x - 2y$ is (______, ______).

k Let $f(x, y) := x - 14y$. *Subject to the constraint that* $y^2 = x$, compute the location (x_0, y_0) of a **global maximum** of f, and compute the location (x_1, y_1) of a **global minimum** of f.
Max=(______, ______); Min=(______, ______).

l Give an example of a *polynomial* f which has a saddle-point at $P := (-5, 2)$.
$f(x, y) :=$ ______.

m Determinant of $\mathsf{M} := \begin{bmatrix} 3 & 4 & 5 \\ 0 & 5 & -1 \\ 0 & 7 & -2 \end{bmatrix}$ is ______.
The characteristic-poly of M is $Ax^3 + Bx^2 + Cx + D$, where $B=$ ______ and $C=$ ______.

n The ***determinant*** of $1{\times}1$ matrix $[z]$ is z. Recursively, the ***determinant*** of $N{\times}N$ matrix

$$\mathsf{B} \quad := \quad \begin{bmatrix} c_{1,1} & c_{1,2} & \dots & c_{1,N} \\ c_{2,1} & c_{2,2} & \dots & c_{2,N} \\ \vdots & \vdots & \dots & \vdots \\ c_{N,1} & c_{N,2} & \dots & c_{N,N} \end{bmatrix}$$

is ______ [Imagine 9 blank lines].

W3: A cylindrical soup-can has radius r and height y; so its volume is $\boldsymbol{\pi} \cdot r^2 \cdot y$. Per square-inch, the metal of the top and the bottom cost thrice as much as the metal of the side. Thus its cost function is

$f(r, y) =$ ______________________________ .

Subject to the volume being held constant (say, 10 cubic inches), what is the *ratio* of y/r that minimizes the cost of the can? Use Lagrange multipliers. Let g denote the specifier fnc that you choose. Compute the three Lagrange equations: [The first eqn. is the constraint eqn.]

C_g **:** = .

L_r **:** = .

L_y **:** = .

Solve the system to compute $\dfrac{y}{r} =$ ______________ .

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.*

W4: Find all points $P = (x, y, z)$ which extremize (maximize or minimize) function $\varphi(P) := -6x + 5y - 3z$, *subject to the condition* that $P = (x, y, z)$ is on the radius-1 **circle** lying in the x,z-plane and centered at the origin. Calling our Lagrange multipliers α and β, write the constraint *function* and *equation* corresponding to α:

$g(P) :=$ ______________________________ .

C_g**:** ______________________________ = ____ .

Write the constraint *function* and *equation* corresponding to β:

$h(P) :=$ ______________________________ .

C_h**:** ______________________________ = ____ .

The three *Lagrange equations* are:

L_x**:** ___ $= \alpha \cdot$ ____________ $+ \beta \cdot$ ____________ .

L_y**:** ___ $= \alpha \cdot$ ____________ $+ \beta \cdot$ ____________ .

L_z**:** ___ $= \alpha \cdot$ ____________ $+ \beta \cdot$ ____________ .

Show all work in setting up and solving equations $(C_g, C_h, L_x, L_y, L_z)$. Please list:

All **max**-points: **(** ___ , ___ , ___ **)** ; ______________________ .

All **min**-points: **(** ___ , ___ , ___ **)** ; ______________________ .

End of Prac-W

W1:	___ ___ ___	180pts
W2:	___ ___	85pts
W3:	___ ___	40pts
W4:	___ ___	85pts
Total:	___ ___ ___	390pts

Honor Code: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ______________________________________