

Due **BoC, Monday, 27Feb2017**, wATMP! Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

W1: *Show no work.*

a **B**-number (Blip number) 22 is **B**-irreducible: $T \quad F$.

B-numbers $K := \dots$ and $N := \dots$ are st. $22 \blacklozenge [K \cdot N]$, yet $22 \nblacklozenge K$ and $22 \nblacklozenge N$. Hence 22 is *not* **B**-prime.

Also, **B**-GCD(175, 70) = \dots .

b On $\Omega := [1..29] \times [1..29]$, define binary-relation **C** by: $(x, \alpha) \mathbf{C} (y, \beta)$ IFF $x \cdot \beta \equiv_{30} y \cdot \alpha$. Statement “*Relation C is an equivalence relation*” is: $T \quad F$

c On \mathbb{Z}_+ , write $x \$ y$ IFF $\text{Gcd}(x, y) \geq 2$. So \$ is Circle
Transitive: $T \quad F$. **Symm.:** $T \quad F$. **Reflex.:** $T \quad F$.

On \mathbb{Z} , say that $x \nabla y$ IFF $x - y < 1$. Then ∇ is:
Trans.: $T \quad F$. **Symm.:** $T \quad F$. **Reflex.:** $T \quad F$.

d On a K -element set, the number of reflexive symmetric binrels is \dots

On a 4-set, there are \dots many equiv.relations.

Carefully TYPE your essays, *double-spaced*. I suggest L^AT_EX, but other systems are ok too.

W2: In our Velleman text, solve problem #12^P.277. Let \mathbf{E}_n be the equilateral triangle with side-length 2^n . This \mathbf{E}_n can be tiled in an obvious way by 4^n many little-triangles [copies of \mathbf{E}_0]; see picture P.277. The “*punctured \mathbf{E}_n* ”, written $\widetilde{\mathbf{E}}_n$, has its topmost copy of \mathbf{E}_0 removed.

A *Trapezoid*, **T**, comprises three copies of \mathbf{E}_0 glued together in a row, rightside-up, upside-down, rightside-up [picture P.277].

i PROVE: *For each n , board $\widetilde{\mathbf{E}}_n$ admits a trapezoid-tiling.* [Call such, a “*zoid-tiling* of $\widetilde{\mathbf{E}}_n$ ”. In a zoid-tiling, all three rotations of the zoid are allowed.]

ii Let Δ_k be the equilateral triangle of sidelength k ; so \mathbf{E}_n is Δ_{2^n} . Triangle Δ_k comprises k^2 little-triangles.
 For what values of k does Δ_k admit a zoid-tiling?
 For which k does $\widetilde{\Delta}_k$ admit a zoid-tiling?

iii An *Lmino* (pron. “ell-mino”) comprises three ■ squares in an “L” shape (all four orientations are allowed).

Let \mathbf{S}_n be the $2^n \times 2^n$ square board, comprising 4^n *squaries* (little squares). Have $\widetilde{\mathbf{S}}_n$ be the board with one corner squarie removed. Velleman inductively shows, pp.272-275, that each \mathbf{S}_n is Lmino-tilable (by $[4^n - 1]/3$ Lminos, of course). Further, with \mathbf{S}'_n denoting \mathbf{S}_n with an *arbitrary* puncture, V. proves that every \mathbf{S}'_n is Lmino-tilable.

Generalize this to three-dimensions. Let \mathbf{C}_n denote the $2^n \times 2^n \times 2^n$ cube, $\widetilde{\mathbf{C}}_n$ the corner-punctured cube, and let \mathbf{C}'_n be \mathbf{C}_n but with an arbitrary *cubie* removed.

What is the 3-dimensional analog of an Lmino? Calling it a “*3-mino*”, how many cubies does it have? [Provide a drawing of your 3-mino.] PROVE: *Every \mathbf{C}'_n admits a 3-mino-tiling.* [Provide also pictures showing your ideas.]

iv *Optional:* Generalize to K -dim(ensional) space, with $\mathbf{C}_{n,K}$ being the $2^n \times \dots \times 2^n$ cube, having $[2^n]^K = 2^{nK}$ many K -dim'al cubies. As before, let $\mathbf{C}'_{n,K}$ be $\mathbf{C}_{n,K}$ with an arbitrary cubie removed.

What is your *K-mino* with which you will tile, and how many cubies does it have? (So a 2-mino is our Lmino.) PROVE: *Every $\mathbf{C}'_{n,K}$ admits a K-mino-tiling.*

W3: [A dodecahedron is a convex polyhedron having 12 faces, 20 vertices and 30 edges; the faces are pentagons.] Two vertices of a regular dodecahedron are *neighbors* if they are *distinct* vertices of a common pentagon. [Each vertex has $[3 \cdot 4] - 3 = 9$ neighbors.] Write $v \sim w$ to indicate that v and w are neighbors. Easily, \sim is symmetric, and anti-reflexive. You can check that \sim is not transitive.

A *labeling* of a regular dodecahedron assigns, to each vertex, a *positive integer*. A labeling is *legal* IFF no pair $v \sim w$ of vertices is assigned the same label.

i Prove there is no legal labeling with vertex-sum [the sum of the 20 labels] equaling 59.

ii Let $\mathcal{S} \subset \mathbb{Z}_+$ be the *set* of vertex-sums obtainable from legal-labelings. Characterize \mathcal{S} explicitly, with proof. You will likely need to construct some particular legal-labelings. [You showed, above, that $\mathcal{S} \not\ni 59$.]

W1: _____ 125pts

W2: _____ 130pts

W3: _____ 95pts

Total: _____ 350pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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