Staple!

Team W____

Sets and Logic
MHF3202 2787

Home-W

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Touch: 4Aug2016

Due **BoC, Monday, 18Nov2013.**, Please *fill-in* every *blank* on this sheet. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

W1: *Show no work, here. Simply fill-in each blank on the problem-sheet.*

___ ___ a Suppose $\mathbf{R}$ is a binrel on set Ω. Then statement "*Relation* $\mathbf{R} \circ \mathbf{R}^{-1}$ *equals* $\mathbf{R}^{-1} \circ \mathbf{R}$" is *T* *F*

___ ___ b Given sets with cardinalities $|B| = 8$ and $|E| = 5$, the number of non-constant fncs in B^E is ______.

___ ___ c Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This* $\approx$ *is an equivalence-relation*" is: *T* *F*

___ ___ d A bijection $f\colon[5,6] \hookrightarrow\!\!\!\!\rightarrow (0,1)$ is: $f(5) :=$ ______;
$f($ ______ $) :=$ ______, for *each* $k \in$ ______;
and $f(x) :=$ ______, for *each* $x \in (5,6] \smallsetminus C$,
where $C :=$ ______.

Your 2 essay(s) must be TYPESET, *and Double or Triple spaced. Use the* Print/Revise *cycle to produce good, well thought out, essays. Start each essay on a* **new** *sheet of paper. Do* **not** *restate the problem; just solve it.*

W2: Prove that the map $f\colon \mathbb{N}\times\mathbb{N} \to \mathbb{Z}_+$, with $f(k,n) := 2^k \cdot [1+2n]$, is injective. Now prove that it is surjective.

W3: The ConsequDay Enigma:

i Over a 30 day month, SeLoidian Bubba posts at least one soln per day, for a total of 42 solns. Prove:

> *There is a period of consecutive days over which he posted exactly* $g := 18$ *solutions.*

[g for "**G**uaranteed".] Note: In your proof, let s_n denote the number of solns posted that month by the end of day n. By hyp., then,

$$1 \leqslant s_1 < s_2 < \ldots < s_{30} = 42\,.$$

Let $t_n := 18 + s_n$. *Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.* [*Hint:* You may find it easier to first show that g=17 is guaranteed. Then you'll see how to show that g=18 is guaranteed.]

ii *Generalize*: Replace 30 by D, replace 42 by P; we now consider posints with D < P. Give a formula for the *largest* value, call it $\Gamma(\mathsf{D},\mathsf{P})$, for which your proof *guarantees* the values $g = 1, 2, 3, \ldots, \Gamma(\mathsf{D},\mathsf{P})$.

iii For fixed D and P, let $\mathcal{M}(\mathsf{D},\mathsf{P})$ be the *set* of guaranteed posints g. What can you tell me about the structure of $\mathcal{M}(\mathsf{D},\mathsf{P})$? Conjectures? Proofs? Computer experiments? (*I* don't know the structure. What can you teach me?)

End of Home-W

W1:	___ ___	95pts
W2:	___ ___	35pts
W3:	___ ___ ___	115pts
Total:	___ ___ ___	245pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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