

Sets and Logic
MHF3202 8768

Home-W

Prof. JLF King
Touch: 4Aug2016

Due **BoC, Monday, 10Feb2014**, Please *fill-in* every *blank* on this sheet. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. *In grammatical English sentences, TYPE your essays on every third line* (usually), *so that I can easily write between the lines. Do not restate the question.*

W1: On a 9x9 chessboard, 37 rooks are placed. Prove there exists a **friendly** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [Hint: PHP] Illustrate the concepts in your proof with *large, useful Pictures*.

W2: Define a sequence $\vec{b} = (b_0, b_1, b_2, \dots)$ by $b_0 := 0$ and $b_1 := 3$ and

†: $b_{n+2} := 7b_{n+1} - 10b_n$, for $n = 0, 1, \dots$

Use induction to prove, for all $k \in [0.. \infty)$, that

‡: $b_k = 5^k - 2^k$.

Further. Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n$, for arbitrary real-number coefficients **S** and **P**?

W3: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

a The number of permutations of "PREPPER" is _____

b The coeff of x^7y^{12} in $[5x + y^3 + 1]^{30}$ is _____

c $\forall x, z \in \mathbb{Z}$ with $x < z$, $\exists y \in \mathbb{Z}$ st.: $x < y < z$. **T F**
 $\forall x, z \in \mathbb{Q}$ with $x \neq z$, $\exists y \in \mathbb{R}$ st.: $x < y < z$. **T F**
 For all sets Ω , there exists a fnc $f: \mathbb{R} \rightarrow \Omega$. **T F**

d Compute the real $\alpha =$ _____ such that

$$3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k = \sum_{j=0}^{1995} \binom{1995}{j} 8^j.$$

[Hint: The Binomial Theorem]

e The number of ways of having 70 objects from 42 types is $\left[\begin{matrix} 70 \\ 42 \end{matrix} \right]_{\text{coeff}}^{\text{Binom}}$ $\binom{\quad}{\quad}$. And $\left[\begin{matrix} 70 \\ 42 \end{matrix} \right] = \left[\begin{matrix} K \\ L \end{matrix} \right]$, where $K =$ _____ $\neq 70$, and $L =$ _____

End of Home-W

W1: _____ 95pts

W2: _____ 75pts

W3: _____ 95pts

Ouch!, scratch work handed-in; OR

Poorly stapled. : _____ -20pts

Total: _____ 265pts

HONOR CODE: "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." *Name/Signature/Ord*

_____ Ord: _____

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_____ Ord: _____