

**Note.** The geometry on  $\mathbb{R}^n$  is defined by the dot-product.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0$ .

**W1:** Show no work.

**a** For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course Circle:

Yes                      True                      Darn tootin'!

**b** In  $\mathbb{R}^3$ , the closest point to  $\mathbf{v} := (1, -7, -5)$  on the line through  $\mathbf{0}$  and  $\mathbf{q} := (1, 2, 3)$ , is  $(\underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}})$ .

In  $\mathbb{R}^2$ , with  $\mathbf{s} := (1, 8)$  and  $\mathbf{w} := (4, -2)$ , compute  $\text{Orth}_{\mathbf{w}}(\mathbf{s}) = \underline{\hspace{1cm}}$ .

**c** Put inner-product  $\langle f, g \rangle := \int_0^2 [f \cdot g]$ . the VS of polynomials. With  $\mathbf{D} := 1 + x$  and  $\mathbf{u} := x^2$ , compute  $\text{Proj}_{\mathbf{D}}(\mathbf{u}) = \underline{\hspace{1cm}}$ .

**d** Let  $R_\theta$  be the std. rotation [by  $\theta$ ] matrix. With  $\mathbf{C} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$  and  $\mathbf{B} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$ , the product  $[\mathbf{CB}]^{35} = \alpha \cdot R_\theta$ , with  $\alpha = \underline{\hspace{1cm}} \in \mathbb{R}_+$  and  $\theta = \underline{\hspace{1cm}} \in (-180^\circ, 180^\circ]$ .

**e**  $\mu = \underline{\hspace{1cm}} \leq \nu = \underline{\hspace{1cm}}$  are the eigenvals of  $\mathbf{G} := \begin{bmatrix} 11 & 30 \\ -6 & -16 \end{bmatrix}$ . Let  $\mathbf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$ . Then  $\mathbf{D} = \mathbf{U}^{-1}\mathbf{G}\mathbf{U}$  where the  $2 \times 2$  integer matrix  $\mathbf{U}$  is  $\mathbf{U} = \left[ \begin{array}{c|c} & \\ \hline & \end{array} \right]$ .

OYOP: Essay: *Write on every **second** line, so that I can easily write between the lines.*

**W2:** Matrix  $\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$ , where  $\mathbf{A}$  and  $\mathbf{D}$  are  $5 \times 5$  and  $7 \times 7$ , resp. Suppose  $\mathbf{C}$  is the  $7 \times 5$  **zero-matrix**. Prove that  $\text{Det}(\mathbf{M}) = \text{Det}(\mathbf{A}) \cdot \text{Det}(\mathbf{D})$ . [*Hint: A good picture helps.*]

**Before** starting your proof, state precisely the formula for determinant that you are using.

End of Class-W

**W1:**                      165pts

**W2:**                 65pts

**Total:**                      230pts

NAME: .....

**HONOR CODE:** *"I have neither requested nor received help on this exam other than from my professor or TA."*

Signature: .....