

W4: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a For $G := \{1, 2, 3, 4\}$, consider $f: G \rightarrow \mathcal{P}(G)$ by

$$\begin{aligned} f(1) &:= \{3, 4\}, & f(2) &:= G, \\ f(3) &:= \emptyset, & f(4) &:= \{1, 4\}. \end{aligned}$$

The set $B := \{x \in G \mid f(x) \not\ni x\}$ is $\{ \dots \}$.

b An explicit bijection $F: \mathbb{N} \times \mathbb{N} \leftrightarrow [-4 .. \infty)$ is

$$F(n, k) := \dots$$

c An explicit bijection $F: \mathbb{N} \leftrightarrow \mathbb{Z}$ is this:

When n is *even*, then $F(n) := \dots$

When n is *odd*, then $F(n) := \dots$

d Both \sim and \bowtie are equiv-relations on a set Ω . Define binrels **I** and **U** on Ω as follows.

Define $\omega \mathbf{U} \lambda$ IFF Either $\omega \sim \lambda$ or $\omega \bowtie \lambda$ [or both].

Define $\omega \mathbf{I} \lambda$ IFF Both $\omega \sim \lambda$ and $\omega \bowtie \lambda$.

So “**U** is an equiv-relation” is: T F

So “**I** is an equiv-relation” is: T F

OYOP: In *grammatical English sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Do not restate the question.

W5: Interval-of-integers $\mathbf{J} := [101 .. 200)$ has 99 elements. A subset $S \subset \mathbf{J}$ is **Big** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is **Perfect** if there exist *distinct* members $x, y \in S$ st. $x + y = 300$.

Prove that **Big** \Rightarrow **Perfect**. [*Hint: PHP. Carefully specify what your pigeon-holes are.*]