Sets and Logic MHF3202 8768 — **Class-W** — Prof. JLF King 31Aug2015

W4:	___ ___	70pts
W5:	___ ___	45pts
W6:	___ ___	45pts
Total:	___ ___ ___	160pts

Please PRINT your name and ordinal. Ta:

Ord: ....

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

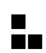
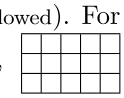
W4: Short answer. Show no work.
Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ a) Sequence $\vec{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 3$, $L_1 := 11$, and $\forall n \in \mathbb{N}: L_{n+2} = -L_{n+1} + 6L_n$. This implies $\forall k \in \mathbb{N}: L_k = [P \cdot \alpha^k + Q \cdot \beta^k]$, for real numbers $\alpha=$ .... $> \beta=$ ...., $P=$ ...., $Q=$ ....

___ b) The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 5 atoms, so I have [expressed as single integer] .... many possibilities.

___ c) Using *only* symbols ***H***, ***D***, $\wedge$, $\vee$, $\neg$, *T*, *F*,], [, rewrite (in simplest form) expression $[[H \Rightarrow D] \Rightarrow H]$ as .... Ditto, rewrite $[H \Rightarrow [D \Rightarrow H]]$ as ....

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Do* **not** *restate the question.* Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...) I suggest you put your name on each sheet.

W5: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{B}_N$ denote the $3 \times N$ board: I.e,  is the $\mathbf{B}_5$ board. Prove:

> *Theorem:* *When N is odd, then board $\mathbf{B}_N$ is not Lmino-tilable.*

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

W6: Interval-of-integers $\mathbf{J} := [101 \,..\, 200)$ has 99 elements. A subset $S \subset \mathbf{J}$ is ***Big*** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is ***Perfect*** if there exist *distinct* members $x, y \in S$ st. $x + y = 300$.

Prove that Big $\Rightarrow$ Perfect. [*Hint:* PHP. *Carefully* specify what your pigeon-holes are.]